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A Simple Method for Predicting the Maximum Squat of a High-Speed Displacement Ship

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Abstract

A simple formula is developed for predicting the maximum squat of a displacement ship as it passes through the transcritical speed range. This is given in terms of a "maximum sinkage coefficient" which is almost constant across a wide range of hull forms. Satisfactory agreement is shown with model test results, and it is shown that large stern sinkages in the order of 3 – 6 metres are predicted for frigate and destroyer type hulls in shallow calm water.

Introduction

Ship squat is the change in vertical position and trim angle of a ship when under way, due to the changed water pressure around the moving ship. At low speeds, the centre of gravity is displaced downwards, and the ship may trim down by the bow (common for full-form ships) or stern (common for fine-form ships). In any case, the available underkeel clearance is usually reduced (compared to the static condition), so that the ship may be at risk of grounding when under way in shallow water.

Squat is very sensitive to ship speed, being approximately proportional to the square of the speed for low ship speeds. Most large ships, such as bulk carriers and containerships, travel in this low-speed range, and hence their squat prediction formulae display a near-quadratic speed dependence (see PIANC 1997 for an overview of low-speed squat formulations).

In shallow water, flow around the ship changes markedly when the ship speed is close to the natural speed of long waves in shallow water (the "critical speed"). Near this speed, wave crests produced by the ship span out almost transverse to the ship's hull, with a generally elevated free surface near the ship's bow and depressed free surface near the stern. Large stern sinkages occur, which are not at all well predicted by the low-speed theories.

The "critical speed" is given by $\sqrt{gh}$, with g the acceleration due to gravity and h the undisturbed water depth (e.g. in a depth of 15m, the critical speed is 12 m/s, or 24 knots). Most large displacement ships are unable to reach this critical speed, due to the large wave resistance at this speed. However, the critical speed could be reached or exceeded by shallow-draft, high-speed ships such as frigates and destroyers. Note that smaller planing or semi-planing hulls are strongly affected by dynamic pressure beneath the hull, and are not the topic of this article. Catamarans also will not be treated in this article, since the important flow interaction between the hulls is still being researched.

For high-speed displacement ships, it has been shown experimentally (Graff et al 1964, Millward & Bevan 1986) and computationally (Chen & Sharma 1995, Gourlay & Tuck 2001) that the midship sinkage reaches a maximum when the ship speed is just below the critical speed, before returning rapidly to near-zero. At the same time, the trim is strongly stern-down, so that the ship's stern suffers a large downward displacement, and may be in danger of grounding. As the critical speed is passed, the midship sinkage and stern sinkage decrease.

Calculations for the maximum predicted midship and stern sinkage in shallow open water were shown in Gourlay & Tuck (2001), based on slender-ship shallow-water theory including wave dispersion. The resulting equations are complicated and difficult to solve numerically, so are not suited to routine use. However, it will be shown here that these equations can be represented in terms of simple "maximum sinkage coefficients" which are almost constant across a wide range of hulls.

Specifically, the maximum midship sinkage $s_{\text{max\_midships}}$ and stern sinkage $s_{\text{max\_stern}}$ will be shown to be represented by

$$
\begin{aligned}
s_{\text{max\_midships}} &= \frac{\nabla}{Lh} C_{\text{max\_midships}} \\
s_{\text{max\_stern}} &= \frac{\nabla}{Lh} C_{\text{max\_stern}}
\end{aligned}
\qquad (1)
$$

where

$\nabla =$ ship's volume displacement

$L =$ ship's waterline length

$h =$ undisturbed water depth

The maximum sinkage coefficients have values

$C_{\text{max\_midships}} \approx 0.5$

$C_{\text{max\_stern}} \approx 2$

over a wide range of hulls.

Origin of the formulae

Theoretical basis

In Gourlay & Tuck (2001), the leading-order effect of dispersion was included in slender-body shallow-water theory, to yield expressions for the sinkage force and trim moment on a ship travelling at transcritical speeds. Here we will use similar expressions, but in a more original form, incorporating the derivative of the section area rather than the section area itself. This allows the method to also be used for transom stern vessels, by representing the flow past the transom as that past an infinitely long cylinder extending downstream from the transom, with cross-section identical to the transom. This method for modelling transom sterns is similar to that used in Tuck et al (2002) for calculating wave resistance of slender ships.

Firstly, Fourier transforms are taken of the rate of change of hull section area dS/dx and waterline breadth distribution $B(x)$.

$$
\begin{aligned}
\overline{B}(k) &= \int_{-L/2}^{L/2} B(x)\, e^{ikx}\, dx \\
\overline{S}_x(k) &= \int_{-L/2}^{L/2} \frac{dS}{dx}\, e^{ikx}\, dx \\
\overline{xB}(k) &= \int_{-L/2}^{L/2} x\, B(x)\, e^{ikx}\, dx
\end{aligned}
\qquad (2)
$$

The waterline length of the ship is L, with the bow at $x = -L/2$ and stern at $x = L/2$.

In a similar manner to Gourlay & Tuck (2001), the vertical force F and bow-up trim moment M (about midships) are given by

$$F = -\frac{\rho g F_h^2}{4\pi}\int_{-\infty}^{\infty}\frac{ik}{\lambda}\,\overline{S}_x(k)\,\overline{B}^*(k)\,dk$$
$$M = \frac{\rho g F_h^2}{4\pi}\int_{-\infty}^{\infty}\frac{ik}{\lambda}\,\overline{S}_x(k)\,\overline{xB}^*(k)\,dk \qquad (3)$$

where

F_h = Froude depth number = (ship speed) / $\sqrt{gh}$

ρ = water density

The asterisk denotes complex conjugate, and λ is found (Gourlay & Tuck 2001) from

$$\lambda = \begin{cases} \sqrt{\left(1-F_h^2\right)k^2 - \frac{h^2}{3}k^4}, & \left(1-F_h^2\right)k^2 - \frac{h^2}{3}k^4 > 0 \\ -i\sqrt{\frac{h^2}{3}k^4 - \left(1-F_h^2\right)k^2}, & \left(1-F_h^2\right)k^2 - \frac{h^2}{3}k^4 < 0 \quad \text{and} \quad k>0 \\ i\sqrt{\frac{h^2}{3}k^4 - \left(1-F_h^2\right)k^2}, & \left(1-F_h^2\right)k^2 - \frac{h^2}{3}k^4 < 0 \quad \text{and} \quad k<0 \end{cases} \qquad (4)$$

Once the vertical force and trim moment are found, the midship sinkage s_{midships} and bow-up trim angle θ can be found by rearranging the hydrostatic equilibrium relations (Gourlay & Tuck 2001) to give

$$s_{\text{midships}} = \frac{1}{\rho g}\left[\frac{M\int_{-L/2}^{L/2} xB\,dx + F\int_{-L/2}^{L/2} x^2 B\,dx}{\int_{-L/2}^{L/2} xB\,dx \int_{-L/2}^{L/2} xB\,dx - \int_{-L/2}^{L/2} B\,dx \int_{-L/2}^{L/2} x^2 B\,dx}\right]$$
$$\theta = \frac{1}{\rho g}\left[\frac{M\int_{-L/2}^{L/2} B\,dx + F\int_{-L/2}^{L/2} xB\,dx}{\int_{-L/2}^{L/2} B\,dx \int_{-L/2}^{L/2} x^2B\,dx - \int_{-L/2}^{L/2} xB\,dx \int_{-L/2}^{L/2} xB\,dx}\right] \qquad (5)$$

This is identical to the alternative method of finding the sinkage directly at the longitudinal centre of floatation (LCF), and taking moments about the LCF to find the trim.

Numerical method

Fourier transforms are calculated using Filon quadrature (Abramowitz & Stegun 1965), which approximates the input function as a parabola over each subinterval, and then integrates the resulting oscillatory integral exactly. The integrands in equation (3) die to zero quickly away from $k = 0$, excepting the singularities at $\lambda = 0$, which must be integrated analytically. More details on the numerical method can be found in Gourlay (2000).

Form of the solution

Figure 1 shows midship sinkage, scaled against waterline length, for the Taylor A3 hull tested by Graff et al (1964) with $h/L = 0.125$.

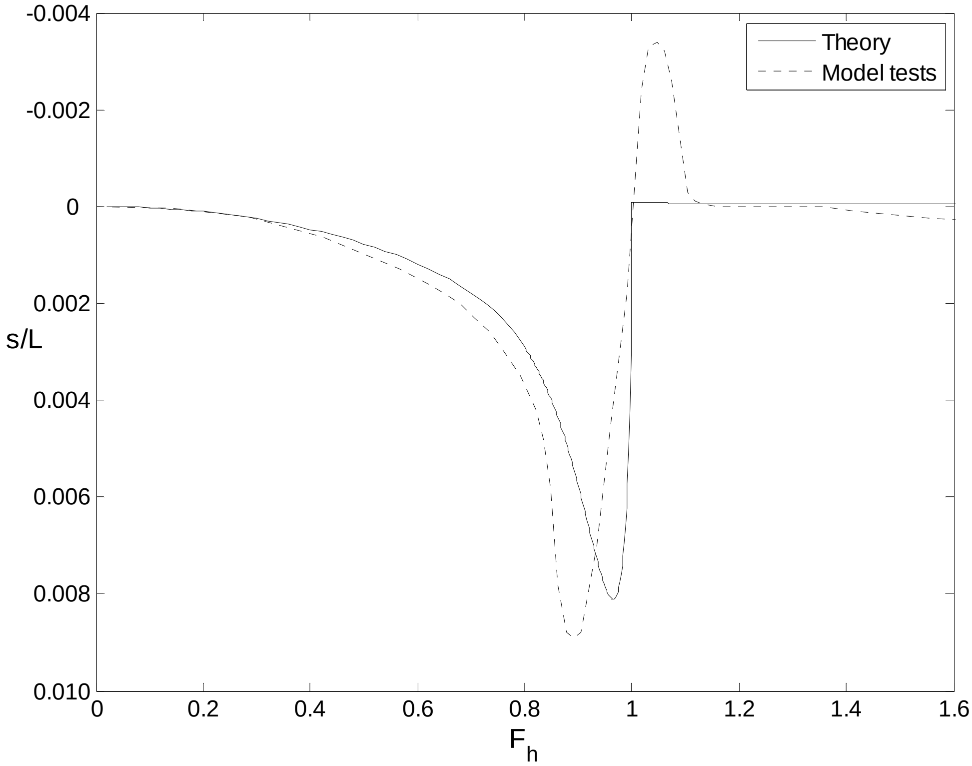

Figure 1: Midship sinkage, scaled against waterline length, for a Taylor A3 hull with $h/L = 0.125$

According to the theory, midship sinkage is roughly proportional to the square of the speed up to $F_h \approx 0.6$, whereupon it increases more quickly and reaches a maximum in the order of 0.8% of the shiplength at $F_h \approx 0.95 - 1.0$.This is followed by a sharp decrease back to near-zero, where it remains at supercritical speeds ($F_h > 1$).

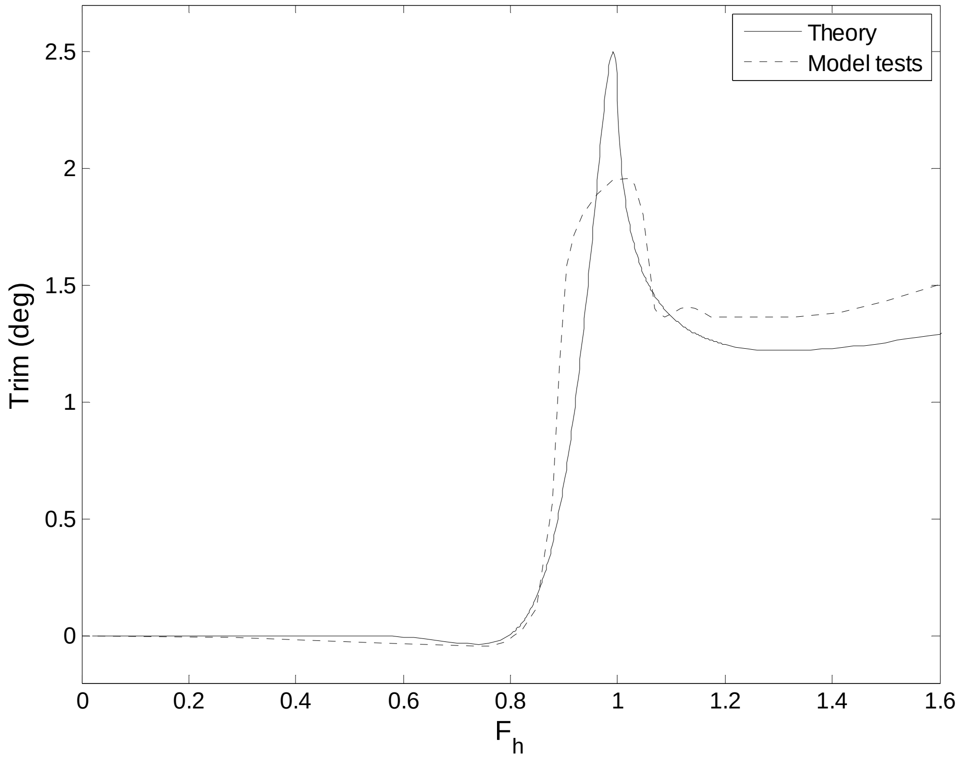

Figure 2: Stern-down trim, scaled against waterline length, for a Taylor A3 hull with $h/L = 0.125$

Stern-down trim is predicted to stay near-zero up to $F_h \approx 0.8$, whereupon it climbs quickly to a maximum of around 2.5° just before $F_h = 1.0$. It then drops down to around 1.2° at higher supercritical speeds.

The main differences in model test results as compared to the theoretical predictions are a maximum sinkage at a lower Froude number, negative sinkage at slightly supercritical speeds, and a broader, smaller trim peak. As discussed in Gourlay & Tuck (2001), transcritical model tests are very sensitive to the presence of sidewalls, since these cause soliton waves to be radiated ahead of the model. All of the differences between theoretical and model test results are explained qualitatively (see Chen & Sharma 1995, Sharma & Chen 2000) by the finite width of the model testing tank used in the experiments (around 36 times the model beam). Therefore it appears likely that full-scale tests in open water would more closely approximate the theoretical predictions.

The stern sinkage is a combination of midship sinkage and stern-down trim, given by

$$s_{\text{stern}} = s_{\text{midships}} + \frac{L}{2}\theta \qquad (6)$$

with θ in radians. For the A3 hull at $h/L = 0.125$, the stern sinkage reaches a maximum of around 2.8% of the waterline length just before $F_h = 1.0$. Note that, since the midship sinkage does not reach a maximum at exactly the same F_h as the trim, the combined stern sinkage has to be plotted separately in order to find the maximum value.

Dimensional analysis

The numerical method described for calculating sinkage and trim through the transcritical speed range is complicated and time-consuming to program. Because of this, it was desirable to express the solution in a dimensionless form, involving coefficients which are only weakly dependent on the hull shape and water depth to waterline length ratio.

We know that the midship sinkage and stern sinkage both reach a sharp peak just before $F_h = 1.0$, and we are primarily interested in the magnitude of these peaks, in order to avoid grounding for a ship passing through the transcritical speed range. An asymptotic analysis was carried out around $F_h = 1.0$, but failed to give an accurate method for finding the peak sinkage. Instead, dimensional analysis was used to find suitable expressions for maximum sinkage, as follows:

QUANTITY	RELEVANT DIMENSION
x	L
k	L^{-1}
$\overline{B}(k)$	$\sim \int_{-L/2}^{L/2} B\, dx = A_{\mathrm{WP}}$ (waterplane area)
$\overline{S}_x(k)$	$\sim \int_{-L/2}^{L/2} \frac{dS}{dx}\, dx \sim \frac{1}{L} \int_{-L/2}^{L/2} S\, dx = \frac{\nabla}{L}$
$\overline{xB}(k)$	$\sim \int_{-L/2}^{L/2} x\, B(x)\, dx \sim L\, A_{\mathrm{WP}}$
F_h	1
λ	$h\, k^2$ near $\left(1 - F_h^2\right) = 0$ from (4)
F (maximum)	$\rho g \frac{1}{h} \frac{\nabla}{L} A_{\mathrm{WP}}$ from (3)
M (maximum)	$\rho g \frac{1}{h} \frac{\nabla}{L} L\, A_{\mathrm{WP}}$ from (3)
$\int_{-L/2}^{L/2} x^2\, B(x)\, dx$	$L^2\, A_{\mathrm{WP}}$
$s_{\mathrm{max\_midships}}$	$\frac{\nabla}{Lh}$ from (5)
θ_{max}	$\frac{\nabla}{L^2 h}$ from (5)
$s_{\mathrm{max\_stern}}$	$\frac{\nabla}{Lh}$ from (6)

Table 1: Dimensional analysis of maximum sinkage

We can therefore write

$$s_{\text{max\_midships}} = \frac{\nabla}{Lh} C_{\text{max\_midships}}$$

$$\theta_{\text{max}} = \frac{\nabla}{L^2 h} C_{\text{max\_theta}} \qquad (7)$$

$$s_{\text{max\_stern}} = \frac{\nabla}{Lh} C_{\text{max\_stern}}$$

introducing the dimensionless coefficients $C_{\text{max\_midships}}$, $C_{\text{max\_theta}}$, $C_{\text{max\_stern}}$.

With the dimensions of the output quantities thus chosen, the coefficients will be unchanged by stretching of the hull in any direction. This is a result of the slender-ship linearization, where the ship's beam and draft are assumed small compared to its length. Fortunately, this is generally true for any displacement ship that intends to travel at transcritical speeds.

Also, since we have found the maxima over a range of Froude numbers, the coefficients are independent of F_h . The coefficients can at most depend on the shape of the hull, and the non-dimensional shallowness h/L (assumed small).

Calculations for example hulls

The maximum sinkage coefficients described above were calculated numerically using equations (5) to calculate s_{midships} , θ and s_{stern} over a range of Froude numbers, for a particular ship in a particular depth of water. The maximum of each of these over the range of Froude numbers gave $s_{\text{max\_midships}}$, θ_{max} and $s_{\text{max\_stern}}$. The numerically-computed coefficients were then found by rearranging equations (7), i.e.

$$C_{\text{max\_midships}} = \frac{Lh}{\nabla} s_{\text{max\_midships}}$$

$$C_{\text{max\_theta}} = \frac{L^2 h}{\nabla} \theta_{\text{max}} \qquad (8)$$

$$C_{\text{max\_stern}} = \frac{Lh}{\nabla} s_{\text{max\_stern}}$$

Hulls tested

The list of high-speed displacement hulls tested numerically is shown in Table 2, along with their block coefficient, longitudinal centre of buoyancy (LCB) and longitudinal centre of floatation (LCF). Note that due to the linearization, calculated coefficients are independent of the size of the vessel, or length/beam or beam/draft ratio. Some of the particulars are calculated from body plans provided and are approximate.

Hull	**Block coefficient**	**LCB aft (% of waterline length)**	**LCF aft (% of waterline length)**
Taylor standard series A3 hull - destroyer type, cruiser stern (Graff et al 1964)	0.59	~0.0 %	~1.8 %
Taylor standard series B5 hull - destroyer type, transom stern (Graff et al 1964)	0.54	~0.4 %	~4.7 %
NPL 100A, 150B hulls - round bilge, transom stern (Millward & Bevan 1986)	0.40	~6.4 %	~8.4 %
High-speed displacement ship series model 1 - fine bow, wide transom (Blok & Beukelman 1984)	0.40	5.0 %	9.2 %
High-speed displacement ship series model 3 - bluff bow, narrow transom (Blok & Beukelman 1984)	0.40	5.2 %	6.8 %
Wigley hull - simple hull with parabolic waterplanes and parabolic sections (Shearer & Cross 1965)	0.44	0.0 %	0.0 %

Table 2: Relevant particulars of hulls to be tested numerically

Computed results

Calculated results for the example hulls are shown in Table 3, for the depth to waterline length ratio $h/L = 0.10$.

Hull	$C_{\max\_midships}$	$C_{\max\_theta}$	$C_{\max\_stern}$
Taylor standard series A3 hull (destroyer type, cruiser stern)	0.62	3.26	2.14
Taylor standard series B5 hull (destroyer type, transom stern)	0.54	2.80	1.82
NPL 100A, 150B hulls (round bilge, transom stern)	0.34	2.30	1.34
High-speed displacement ship series model 1 (fine bow, wide transom)	0.40	2.57	1.54
High-speed displacement ship series model 3 (bluff bow, narrow transom)	0.40	2.68	1.61
Wigley hull (simple hull with parabolic waterplanes and parabolic sections)	0.62	3.06	2.05

Table 3: Calculated maximum sinkage coefficients for example hulls

In all cases it was found that the maximum midship sinkage and stern sinkage both occur in the range $0.95 < F_h < 1.0$. A sensitivity analysis was performed for different values of h/L, but the coefficients were found to be approximately constant over the range of interest $0.05 < h/L < 0.15$. Note that shallow-water theory loses its validity at larger values of h/L, but in that case a slender ship is not at risk of grounding.

Comparison with experimental results

To date, the author has found no available full-scale data on combined sinkage and trim of displacement ships through the transcritical speed range in open water. Many model tests have been performed at transcritical speeds, but often the width of the tank is not large enough to prevent solitons being radiated. Tank walls have a significant effect on the flow at transcritical speeds in shallow water, even when the tank width is 20-30 times the model beam. Even without sidewall effect, the large difference in Reynolds numbers may cause a discrepancy between model-scale and full-scale trim.

Two sets of model tests will be referred to here. The first (Graff et al 1964) tested the Taylor series A3 and B5 hulls. The second (Millward & Bevan 1986) tested the NPL 100A and 150B hulls (among others). In Graff et al (1964) the tank was around 36 times the models' beam, and in Millward & Bevan (1986) it was around 20 times the models' beam. Note that both sets of tests used a towed rather than a self-propelled model, which is a further source of discrepancy (mainly for trim) when scaling to full scale self-propelled ships.

Results have been compared in terms of the sinkage coefficients, defined by equations (8). The comparisons are shown in Table 4.

Hull	$c_\forall = \dfrac{\nabla}{L^3}$	h/L		$C_{\text{max\_midships}}$	$C_{\text{max\_theta}}$	$C_{\text{max\_stern}}$
Taylor A3	0.0017	0.125	calculated	0.60	3.24	2.12
			experimental	0.65	2.50	1.68
Taylor B5	0.0017	0.125	calculated	0.53	2.78	1.80
			experimental	0.54	1.93	1.36
NPL 100A	0.0035	0.167	calculated	0.30	2.25	1.28
			experimental	0.43	2.29	1.19
NPL 150B	0.0052	0.167	calculated	0.30	2.25	1.28
			experimental	0.41	2.03	1.09

Table 4: Comparison of theory with model test results in a wide tank

Overall, there is satisfactory agreement between the predicted and experimental results. The following points are noted:

- The magnitude of midships sinkage shows good agreement with the results of Graff et al (1964). The A3 and B5 hulls are ideal for the theoretical method used, having a low volumetric coefficient $c_\forall$ (i.e. high length/beam and length/draft ratios).
- The maximum trim reported in Graff et al (1964) is significantly less than that predicted. As discussed in Gourlay & Tuck (2001), numerical results for a ship in a channel (Chen & Sharma 1995) show that channel walls have the effect of decreasing the maximum trim. Therefore true open-water test results would likely show better agreement than these finite-width test results.
- The maximum midship sinkage of the NPL hulls is underpredicted by the theory. Gourlay & Tuck (2001) showed that the theory underpredicts midship sinkage at large values of h/L such as this; it is more appropriate to use a finite-depth rather than shallow-water theory in this case. $h/L = 0.167$ was the shallowest depth for which data were available, but is outside the range of depths for which the ship is

at risk of grounding. In addition, the NPL hulls have a large volumetric coefficient and are pushing the limits of slender-ship theory.

- The theory's close prediction of maximum trim of the NPL hulls is most likely coincidental; the experimental trim is likely to be reduced due to sidewall effect, and the predicted trim is likely to be reduced due to the large value of h/L.
- The Froude depth number at which the maximum midship sinkage occurred was around 0.89 for the Graff et al (1964) results, and around 0.85 for the Millward and Bevan (1986) results. This is lower than the predicted range of 0.95-1.0. Sharma & Chen (2000) showed that the presence of sidewalls will tend to decrease the Froude depth number at which the maximum sinkage occurs.

Generalizations and examples

An analysis of the maximum sinkage coefficients shows that the maximum midship sinkage coefficient shows some correlation with LCB, while the maximum trim coefficient, and hence maximum stern sinkage coefficient, correlates with LCF. Little correlation is found with block coefficient, since this is effectively scaled out in the dimensional analysis. Length/beam and length/draft ratios are also scaled out through the linearization, provided the ship is slender.

Therefore the following guidelines are offered for choosing coefficients for general high-speed displacement hulls. These are given in terms of the position of LCB and LCF (% of waterline length). Note that transom stern vessels, with LCB and LCF further aft, will tend to have smaller maximum sinkages than cruiser stern vessels.

$$C_{\text{max\_midships}} = \begin{cases} 0.6 & \text{LCB} = 0-4\% \text{ aft} \\ 0.4, & \text{LCB} = 4-8\% \text{ aft} \end{cases}$$
$$C_{\text{max\_stern}} = \begin{cases} 2.0 & \text{LCF} = 0-5\% \text{ aft} \\ 1.5, & \text{LCF} = 5-10\% \text{ aft} \end{cases} \tag{9}$$

Maximum midship and stern sinkage are then given by

$$s_{\text{max\_midships}} = \frac{\nabla}{Lh} C_{\text{max\_midships}}$$
$$s_{\text{max\_stern}} = \frac{\nabla}{Lh} C_{\text{max\_stern}} \tag{10}$$

Clearly it is the maximum stern sinkage which is the limiting factor in avoiding ship grounding at transcritical speeds.

As an example, maximum sinkages have been estimated for a Perry-class frigate (FFG7) and Spruance-class destroyer (DD963V) using the above formulae. Relevant particulars of these hulls are shown in Table 5. Stern drafts include appendages.

Hull	Length btw perpendiculars	Displacement	Volume displacement (salt water)	Draft midships	Draft stern	LCB	LCF
FFG7	124.4m	4080 tonnes	3980 m^3	4.8m	7.0m	2.0% aft	5.9% aft
DD963V	161.2m	9040 tonnes	8820 m^3	6.5m	9.0m	2.2% aft	8.0% aft

Table 5: Relevant particulars of Perry-class frigate (FFG7) and Spruance-class destroyer (DD963V)

According to equation (9), the correct coefficients to use for both of these vessels are $C_{\text{max\_midships}} = 0.6$ and $C_{\text{max\_stern}} = 1.5$.

Calculated maximum sinkages and dynamic drafts for the Perry-class frigate in different water depths are shown in Table 6. Dynamic draft is defined as the static draft plus the downward sinkage. Maximum sinkage occurs at just below the critical speed.

Water depth (h)	Critical speed	$s_{\text{max\_midships}}$	$s_{\text{max\_stern}}$	Dynamic draft midships	Dynamic draft stern	Underkeel clearance
13.0m	22.0 kts	1.5m	3.7m	6.3m	10.7m	2.3m
12.0m	21.1 kts	1.6m	4.0m	6.4m	11.0m	1.0m
11.0m	20.2 kts	1.7m	4.4m	6.5m	11.4m	-0.4m

Table 6: Example calculations for Perry-class frigate (FFG7)

Calculated maximum sinkages and dynamic drafts for the Spruance-class destroyer in different water depths are shown in Table 7.

Water depth (h)	Critical speed	$s_{\text{max\_midships}}$	$s_{\text{max\_stern}}$	Dynamic draft midships	Dynamic draft stern	Underkeel clearance
16.0m	24.4 kts	2.1m	5.1m	8.6m	14.1m	1.9m
15.0m	23.6 kts	2.2m	5.5m	8.7m	14.5m	0.5m
14.0m	22.8 kts	2.3m	5.9m	8.8m	14.9m	-0.9m

Table 7: Example calculations for Spruance-class destroyer (DD963V)

We see that large stern sinkages in the order of 3 – 6 metres are predicted for the frigate and destroyer while passing through the critical speed. The negative underkeel clearances for the frigate in 11m water depth, or the destroyer in 14m water depth, suggest that the ships will not be able to pass through the transcritical speed in these water depths without being at significant risk of grounding.

Conclusions

Simple expressions have been put forward for predicting the maximum midship sinkage and stern sinkage of a high-speed displacement ship in shallow open water. These expressions depend only on the displacement, waterline length and water depth, with coefficients that are weakly dependent on the shape of the ship (principally LCB and LCF).

Verification is difficult at this stage, since full-scale data are not easily available, and model scale data are affected by the presence of sidewalls, even in wide tanks. These formulae should be used with caution until further verification is possible.

Nevertheless, the formulae provide useful guidelines for estimating grounding risk at high speed in shallow water. They also highlight the drastic nature of flow around a ship at transcritical speeds, with large stern sinkage and significant grounding risk.

Provided the general dimensional nature of the solution is correct, coefficients can be adjusted in future as more experimental data becomes available.

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