14th Australasian Fluid Mechanics Conference
Adelaide University, Adelaide, Australia
10-14 December 2001

Wave Forces on a Body in Confined Waters

J. Xia[1] and J.R. Krokstad[1,2]

[1]Centre for Marine Science and Technology
Curtin University of Technology, Perth, WA 6845, AUSTRALIA

[2]Division of Offshore Structures
Norwegian Marine Technology Research Institute, Trondheim, N-7450, NORWAY

Abstract

In this paper, we study the interaction of gravitational water waves with stationary ocean structures near a vertical boundary or in a wave channel. The water depth is assumed finite implying that the interference effect of not only the vertical boundaries but also the horizontal bottom is considered. Presented here are a potential flow theory and the boundary-element method (BEM) towards solving this problem. The numerical results include 1st- and 2nd-order wave exciting forces of a hemisphere, and are compared with available computations and published model test results. The theoretical findings based on a parametric study in this paper are associated with practical design considerations for a proposed deepwater ocean basin to be built at Jervoise Bay, Western Australia.

Introduction

In many ocean engineering applications, there is a need to study wave interaction with a body or body system, which is affected by the surrounding boundaries of the wave field. For example, the motion behaviour of a vessel in a wave channel or close to a quay may be quite different from the same vessel in the open sea.

Part of the motivation for this study is related to a proposed deepwater ocean basin to be built in Western Australia. It is proposed that the basin will have a horizontal operational area of 50*m* x 50*m*. The maximum depth is 20*m* plus a 15*m* pit. The basin will be equipped with multi-flap wave generators on two adjoining sides. On the opposite sides of the wave generators, absorbing beaches or wave diffusers may be fitted. We need to answer questions about the horizontal dimensions, the necessity to install wave absorbing beaches and active wave absorption on wave generators and the necessity of using a removable bottom for water depth change *etc*. For instance, when conducting model tests by using only one side of vertical wave generators, the other side will perform as a reflecting wall if no active wave absorption is attempted. Thus, wall effects cannot be totally neglected, even in a wide ocean basin designed to avoid significant wave reflections from the basin boundaries. This paper attempts to shed light on these questions by numerically modelling the wave forces on a surface-piercing body in wave tanks of various configurations and comparing the results with open-sea situations.

Potential theory is often used to describe hydrodynamic wave-body interaction problems when the wavelength is not much longer than the body dimensions. For arbitrary-shaped bodies, the boundary-element method based on Green function approaches is a powerful numerical method for solving the problems.

Successful computation of wave-body interactions is to a large extent dependent on efficient evaluation of the Green function. Newman [6], among others, has developed efficient algorithms for some of the Green functions for open-sea problems, which resulted in practically applicable boundary-element method computer programs such as WAMIT [4]. The Green function for a horizontally semi-infinite wave field is the sum of the Green function for the open sea and its image about the vertical boundary, which does not add in too much computational burden. However, a Green function that models wave-body interactions in a wave channel with two parallel vertical walls, such as a towing tank, introduces far more complexity. In this case, the Green function satisfies the linearized free-surface condition and the non-penetration condition on a flat bottom and the sidewalls. It may be formally represented by an infinite series of mirror images of the Green function in the open-sea. The convergence of such a series, however, is known to be extremely slow.

In order to effectively model the channel problems, many efforts have been made towards accurate and efficient computation of the Green function; see for example, [1, 3, 5, 7, 9]. In the recent advancement made by Xia [9], a consistent asymptotic analysis has been developed for fast evaluation of the complete image series of the open-sea Green function. The numerical scheme was successfully applied to the investigation of the wave-interference effects on a truncated cylinder in a channel and was verified with a semi-analytical study published in [12]. Xia and Ronalds [10] presented numerical results for a FPSO hull in a wave tank and compared them with the numerical solution in open-sea.

In what follows, we will present an introduction to the potential theory and the boundary-element method for wave-body interactions in an open-sea, near a vertical wall and in a wave tank of parallel sidewalls. Computational results for the first-order wave exciting force in heave and the second-order mean drift force in surge of a hemisphere in a wave tank with parallel sidewalls will be presented and be compared with model testing results published in [13]. These results clearly demonstrate the sidewall effects in typical offshore hydrodynamic modelling by a towing tank and the robustness of the present numerical approach. The numerical method will then be applied to a parametric study to investigate the effects of a single reflecting wall and two parallel reflecting walls on wave force modelling of the hemisphere. Associated discussions will be given towards the design and operation issues of the proposed Australian ocean basin.

Boundary Element Method

Potential Flow Theory

We consider the problem of a three-dimensional body of arbitrary geometry subjected to regular waves in a wave basin of 'infinite' length and width b. The water depth in the basin is denoted by h. Cartesian coordinate system *o-xyz* is defined with *x*-axis along the 'longitudinal' direction towards the absorbing beach and coincident with the centre line of the basin, *y*-axis pointing to the side wave maker, *z*-axis positive upwards and the origin *o* placed on the undisturbed free surface. A regular wave of circular frequency ω sent by the end wave maker propagates in the positive *x*-direction. The body is assumed fixed in the water.

Under the assumptions of a perfect fluid and irrotational flow, the motion of the fluid field can be described in terms of velocity potential in the form

$$\Phi(x,y,z,t) = \mathrm{Re}\{\phi(x,y,z)e^{-i\omega t}\} \tag{1}$$

with the spatial potential

$$\phi(x,y,z) = \frac{gA}{i\omega}(\phi_I + \phi_D) \tag{2}$$

where A is the incident-wave amplitude; ϕ_I and ϕ_D are normalized incident and diffracted wave potentials.

The incident-wave potential ϕ_I may be explicitly written as

$$\phi_I = \frac{\cosh k(z+h)}{\cosh kh} e^{ikx} \tag{3}$$

where k is the wave number satisfying the dispersion relation $\omega^2 = gk \tanh kh$, with g being the gravitational acceleration.

The governing equation for the diffraction potential ϕ_D is the Laplace's equation subject to the linearized free-surface boundary condition and the non-penetration boundary condition on the seabed, on the vertical walls $y = b_1, b_2$ that model the sidewalls of a wave tank and on the body surface S_0,

$$\begin{cases} \left(\dfrac{\partial^2}{\partial^2 x} + \dfrac{\partial^2}{\partial^2 y} + \dfrac{\partial^2}{\partial^2 z}\right)\phi_D = 0 & \text{in the fluid} \\ -\omega^2 \phi_D + g\dfrac{\partial \phi_D}{\partial z} = 0 & \text{on } z = 0 \\ \dfrac{\partial \phi_D}{\partial z} = 0 & \text{on } z = -h \\ \dfrac{\partial \phi_D}{\partial y} = 0 & \text{on } y = b_1, b_2 \\ \dfrac{\partial \phi_D}{\partial n} = -\dfrac{\partial \phi_I}{\partial n} & \text{on } S_0 \end{cases} \tag{4}$$

where n denotes the unit inward normal to the mean underwater body surface S_0 with three Cartesian components (n_1, n_2, n_3) and the extended definition, $(n_4, n_5, n_6) = (yn_3 - zn_2, zn_1 - xn_3, xn_2 - yn_1)$. In addition, the diffraction potential must satisfy a radiation condition that states that, when $x \to \pm\infty$, ϕ_D is associated only with waves that propagate away from the body.

For an open-sea problem, the boundary condition on the vertical walls may be removed by assuming $b_1 = -\infty$ and $b_2 = +\infty$ and replacing the associated non-penetration condition by the radiation condition. Accordingly, we may define $b_1 = -\infty$, $b_2 = b/2$ for a quay or a single tank wall and $b_1 = -b/2$, $b_2 = b/2$ for a wave tank with two parallel reflecting walls.

Based on the linearized Bernoulli's equation, the amplitude of the first-order wave exciting force (and moment) is represented by the velocity potentials as

$$\boldsymbol{F}^{(1)} = \rho g A \iint_{S_0} (\phi_I + \phi_D)\boldsymbol{n} dS \tag{5}$$

where $\boldsymbol{n} = (n_1, n_2, \cdots, n_6)$ and ρ is the fluid density.

The mean second-order fluid forces (and moments) for a fixed surface-piercing body are defined as the mean over a complete wave cycle and can be expressed in terms of the first-order velocity potential as [10]

$$\begin{aligned} \boldsymbol{F}^{(2)} &= \frac{1}{2g}\rho\omega \overline{\int_{L_W} \Phi_t^2 \sec\alpha\, \boldsymbol{n} dl} - \frac{1}{2}\rho \overline{\iint_{S_0} |\nabla\Phi|^2 \boldsymbol{n} dS} \\ &= \frac{1}{4g}\rho\omega^2 \int_{L_W} |\phi|^2 \sec\alpha\, \boldsymbol{n} dl \\ &\quad - \frac{1}{4}\rho \iint_{S_0} (|\phi_x|^2 + |\phi_y|^2 + |\phi_z|^2)\boldsymbol{n} dS \end{aligned} \tag{6}$$

where L_W is the mean waterline and α, the flare angle (zeroed for vertical sides).

Green function approach

In order to find a solution to the diffraction velocity potential, we may employ the Green function, $G(p,q)$, which satisfies the entire boundary conditions except that for the body surface. It represents the spatial part of the velocity potential at a field point, $p = (x,y,z)$, in the wave tank due to a pulsating source of unit strength at the point $q = (x', y', z')$. To eliminate the detrimental effect of 'irregular frequencies', we introduce an interior fluid domain, which is enclosed by the body surface S_0 and an artificial flat top S_a. By applying Green's theorem respectively to the volume of fluid in the exterior and interior domain, an extended source representation of the potential may be derived as (see for example, [11])

$$\phi_D(p) = \iint_{S_0 + S_a} \sigma(q) G(p,q) dS_q \tag{7}$$

The above equation is valid for field point *p* on the body surface and within the exterior or interior fluid domain. When the fluid velocity is specified on the body surface and its artificial top for the exterior problem, a pair of integral equations for the source strength is obtained from the normal derivative of (7), which forms the basis for a boundary-element method for numerically solving the velocity potential. The boundary-element method is efficient provided the Green function and its derivatives are evaluated accurately and efficiently.

The Green function of the boundary-value problem (4) must satisfy the same boundary conditions except that the body boundary condition on S_0 can be ignored. The governing Laplace's equation is replaced by

$$\nabla^2 G(p,q) = -4\pi\,\delta(p-q) \qquad \text{in the fluid} \tag{8}$$

where δ is the Dirac delta function. As for the velocity potential, the Green function must also satisfy a radiation condition that states that, at infinity, *G* is associated only with waves that propagate away from the source.

For the open-sea problem *i.e.* $b_1 = -\infty$ and $b_2 = +\infty$ the Green function is denoted by

$$G = G_0 \qquad (9)$$

and is given in [8] in the form of principal-value integral and in [2] in the series form know as John's series. Direct computation of the open-sea Green function based on the integral form and the John's series is inefficient and impractical. Many careful treatments have been developed for accurate and efficient evaluation of the open-sea Green function (*e.g.* [6]).

Lets consider the images of the source at $q_m = (x', y'_m, z')$, where y'_m is defined by

$$y'_m = (-1)^m y' + mb \qquad (10)$$

and write the open-sea Green function of the *m*-th image as

$$G_m = G(p, q_m) \qquad (11)$$

It is noted that q_0 represents the source itself and (11) reduces to (9) when $m = 0$.

For a single wall problem with $b_1 = -\infty$, $b_2 = b/2$ the Green function is the sum of the open-sea Green function and its image about the wall,

$$G = G_0 + G_1 \qquad (12)$$

while for a narrow tank problem with two parallel walls at $b_1 = -b/2$, $b_2 = b/2$, the Green function should represent the potential induced by an open-sea source and its infinite number of mirror images,

$$G = \sum_{m=-\infty}^{\infty} G_m \qquad (13)$$

The evaluation of the slowly convergent open-sea Green function series (13) involves many difficulties. A recent approach has been given in [9], which forms the basis for part of the following numerical investigations.

Numerical Results and Discussions

Numerical and experimental results of 1st- and 2nd-order wave forces presented hereafter are made for a body with its under water part being a hemisphere. The radius of the hemisphere is denoted by *r* and diameter, *d*. The body surface is modelled by 256 panels with 8 intervals in the depth direction and 32 in the circumferential direction. For a surface-piercing body in an open-sea of finite water depth, we may validate the present numerical calculations with the widely accepted computer program WAMIT. The present computations of 1st-order vertical wave force and 2nd-order horizontal mean drift force at three water-depth/draught ratios (1.2, 2 and 20) agree very well with those results from WAMIT. In what follows, we provide numerical results and discussions on the wall effects.

a) <u>Effects of Two Parallel Walls</u>

The hemisphere is placed in the centre of a wave tank with two parallel reflecting walls in the incident wave direction. The water depth is $10d$. Figures 1 and 2 compare the present numerical results with the experimental data obtained at the Norwegian Marine Technology Research Institute (MARINTEK) [13]. The measurement was done for a hemisphere of 1 *m* in diameter in a towing tank of 10.5 *m* wide and 10 *m* deep. Very good agreement is observed here between the calculated and measured 1st-order wave force in heave; both well capture the tank wave resonance that occurs when the tank-width/wavelength ratio approaches an integer. Measurement of 2nd-order forces in a large sloshing wave tank is a very difficult task. Nevertheless, Figure 2 shows encouraging agreement between measured and calculated 2nd-order mean drift force with strong tank wall effects.

Figures 3 and 4 present the calculated 1st-order heave force and 2nd-order mean drift force for the hemisphere in the wave tank at different width. It is found that extending the transverse surface space from a narrow towing tank to a wide ocean basin will only moderately reduce the tank wall effect on, in particularly, the 2nd-order drift force assuming 100 % reflection from two opposite wall. In this case, a significant wall effect can still be observed when $b/d = 21$ corresponding to a hemisphere of diameter 2.38 *m* in a wave basin 50 *m* in width. Generally speaking, the wall effect on the 2nd-order drift force is found to be stronger than on the 1st-order heave force.

b) <u>Effects of One Vertical Wall</u>

Consider now the hemisphere stationed near a vertical reflecting wall. This corresponds to the situation when running just one side of wave generators in an ocean basin with the other acting as a 100 % reflector. The distance of the body centre to the wall is $b/2$ and the water depth $h = 10d$. The incident wave direction is parallel to the wall. Figures 5 and 6 respectively illustrate the calculated 1st-order wave force in heave and the 2nd-order mean drift force in the wave propagation direction. According to the present computation and its comparison with the open-sea result, it is seen that the wall effect on the 1st-order heave force is limited, whereas the wall significantly affects the 2nd-order drift force. From Figure 6, it may be found that resonance occurs when the ratio of the distance between the body and the wall to half wavelength is approximately an integer. By increasing the distance to the wall, the wall effect decreases. However, an obvious wall effect is observed when $b/d = 21$ corresponding to a hemisphere of diameter 2.38 *m* in the centre of a wave basin of 50 *m* in width. The results from this example indicate that reflection effects for a single wall problem are far less important than for two opposite walls but still can be important for 2nd-order forces.

Conclusions

The extension of the traditional BEM for floating bodies in open wave fields to confined waters has been confirmed to be successful. This is based on an accurate and efficient evaluation of the Green functions formulated using the *method of images*. The application of the new numerical approach to the parametric investigation of tank wall effects on wave force modelling of a hemisphere offers quantitative information for the design and operation of an ocean basin. It is found that the horizontal 2nd-order wave drift force on a surface-piercing body is sensitively affected by the wave reflections from either a single vertical wall or two parallel reflecting walls.

Acknowledgments

The financial support from the ARC Small Grant scheme, the WA Department of Commerce and Trade, MARINTEK, Invest Australia, Woodside and BHPP is gratefully acknowledged.

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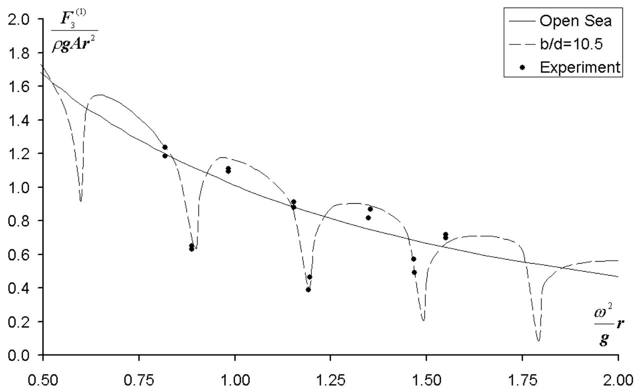

Figure 1. Calculated 1st-order heave force in open-sea and in a towing tank compared with measurement [13]

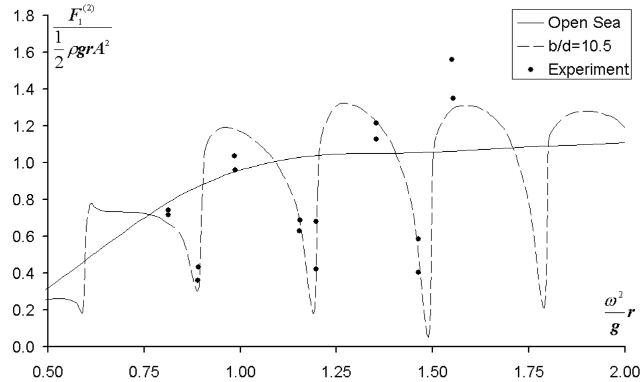

Figure 2. Calculated 2nd-order surge force in open-sea and in a towing tank compared with measurement [13]

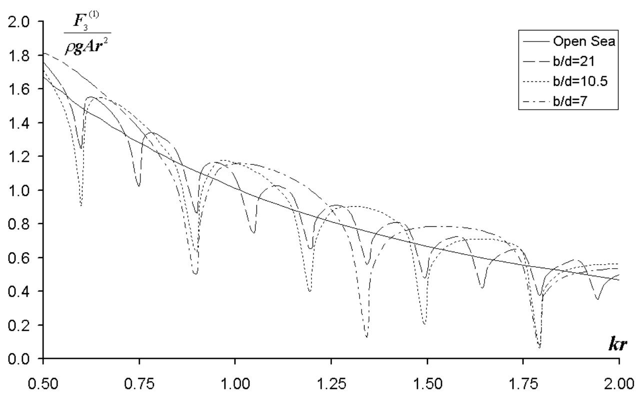

Figure 3. Calculated 1st-order heave force in a wave tank of different width compared with open-sea case

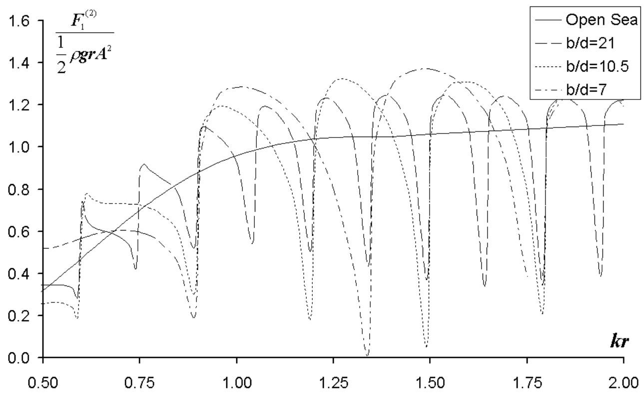

Figure 4. Calculated 2nd-order surge force in a wave tank of different width compared with open-sea case

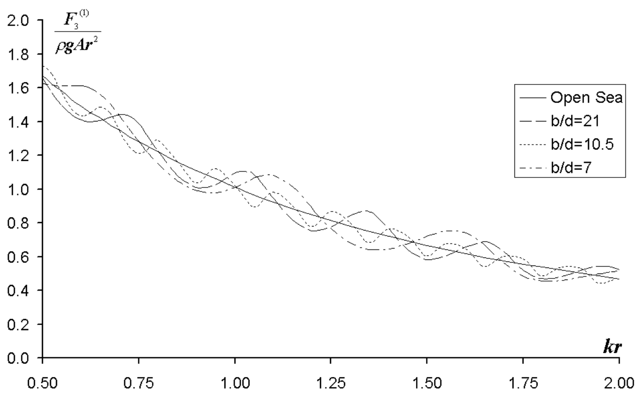

Figure 5. Calculated 1st-order heave force for the case of single wall with changing distance to the body

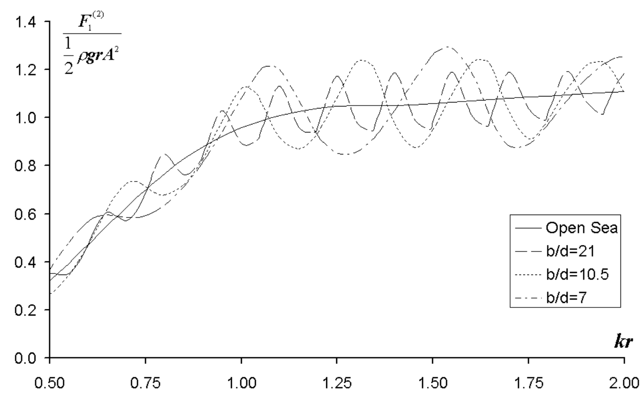

Figure 6. Calculated 2nd-order surge force for the case of single wall with changing distance to the body