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A brief history of mathematical ship squat prediction, focussing on the contributions of E.O. Tuck

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Abstract This article traces the contributions of Prof. E.O. Tuck to the field of mathematical ship squat prediction. The review expands on Tuck's own review of his early work [1] and describes the use of his formulae in modern squat prediction methods. A method for calculating Tuck's sinkage and trim coefficients using easily-obtainable ship parameters is also described.

Keywords E.O. Tuck, ship squat, slender-body shallow-water theory, sinkage and trim coefficients

Nomenclature

A_{WP}	ship waterplane area
$A_{\mathrm{fwd,aft}}$	waterplane area forward or aft of parallel midbody
$B(x)$	ship waterline breadth at station x
B_{m}	ship maximum waterline breadth
F_h	depth-based Froude number $U/\sqrt{gh}$
g	acceleration due to gravity
h	water depth
I_{LCF}	second moment of waterplane area $\int_{-L/2}^{L/2}(x-\mathrm{LCF})^2 B\,dx$
L	ship submerged length
LCB	Longitudinal Centre of Buoyancy
LCF	Longitudinal Centre of Floatation
s	sinkage
$S(x)$	ship submerged cross-sectional area at station x
T	ship draught
U	ship speed
w	canal width
x	longitudinal coordinate, centred at submerged midships, positive aft
$x_{\mathrm{fwd,aft}}$	position of forward or aft extremity of parallel midbody
y	transverse coordinate, centred on ship centreline, positive to starboard
z	vertical coordinate, centred at undisturbed free surface, positive upwards
$\alpha_{\mathrm{fwd,aft}}$	longitudinal scaling factors forward or aft of parallel midbody
ϕ	disturbance velocity potential
θ	dynamic trim angle (radians), positive bow-down
$\xi_{\mathrm{fwd,aft}}$	LCF of hull section forward or aft of parallel midbody
∇	ship submerged volume

1 Introduction: ship squat and under-keel clearance

Ship squat is defined as the change in a ship's vertical position when under way. It is normally characterized by a bodily sinkage and a dynamic change in trim. Therefore both the bow and stern normally sink further downwards as the speed increases, but by different amounts. For large modern bulk carriers or containerships, bow and stern sinkage can be in the order of 1 – 2 metres. This may cause the ship to run aground if it is moving too fast in shallow water. The effect is shown in Figure 1.

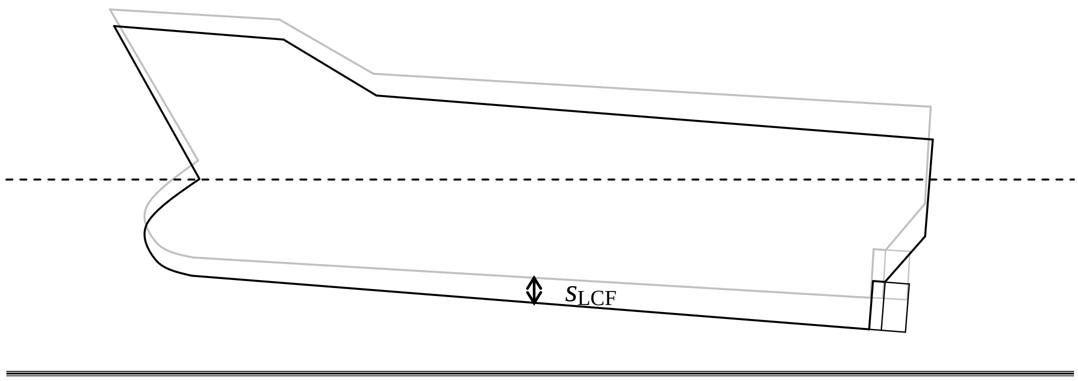

Figure 1: Ship in static floating position (grey outline) and under way (black outline). Dashed line shows undisturbed water level. Sinkage at LCF is s_{LCF} . Change in trim angle is θ (positive bow-down)

In the 1930s, sinkage of model ships had been measured and scaled to full scale by Horn [2]. Around the same time, sinkage of an ellipsoid had been calculated by Havelock [3], assuming potential flow. They found that, for moderate speeds of moderate size ships, ship sinkage was in the order of a decimetre. Mariners however could not measure the bodily sinkage of a ship at sea, due to the lack of a defined vertical reference. Furthermore, what the mariners used as a visual reference, i.e. the sea level around the ship, was misleading them, since the free surface around a moving ship is also pulled bodily downwards. Apart from a few dedicated model tests, most early model tests focused on resistance and used a towed model; there seemed no need to set up a vertical reference to assess the bodily sinkage of the model. Therefore up until the early 1960s, the sinkage component of ship squat was mostly ignored in both model testing and theoretical investigations.

It was known through seamen's experience, as well as model tests [4] that trim could change significantly at certain speeds in shallow water. This effect could be measured by mariners with a pendulum on board the ship, and was seen to be important, as a large bow-up trim correlated with a large resistance and hence fuel usage. The effect of trim on squat must also have been recognized; for example, an observed trim change of 2ft between the bow and stern when underway could be used to conclude that the bow had sunk by 1ft and the stern had risen by 1ft. However in most cases, trim and its effect on squat were small, especially for the ships of the day which were mostly close to fore-aft symmetric.

The 1960s saw unprecedented growth in the size of cargo ships being brought into service. Ports that had up until this time been considered deep water, were suddenly considered shallow water. Questions were inevitably asked about whether the vertical position of a ship, and hence the clearance between a ship's keel and the seabed, was affected by the ship's speed through the water. Due to the limited information available, rules of thumb were developed for under-keel clearance, such as “allow one foot of clearance for squat” or “allow one foot for every 5 knots of speed” [5].

Fortunately for the shipping world, both the model testing community and mathematical community came to the rescue before any of the new large ships suffered a serious grounding. Model testing tanks started measuring bow and stern sinkage [6], using a towing carriage as a vertical reference, and the results were scaled up to full scale. However, a physical theory of ship squat was urgently needed in order to make sense of the model test data and be able to predict the squat of general ships at general speeds.

2 The foundations of ship squat theory

As far as the need for a physical squat theory was concerned, Ernie Tuck was in the right place at the right time, with the right skills. Having completed his PhD thesis “The steady motion of a slender ship” [7] at Cambridge, he arrived at the David Taylor Model Basin in 1963. Here he was able to continue his theoretical treatment of slender ships, but all the while be surrounded by model ship testing and the practicalities of ship design. In 1964 he published an asymptotic expansion [8] for the flow around a slender ship in deep water, in which the technique of matched asymptotics was used to define the leading-order approximation to the kinematic hull boundary condition.

1964 also saw the publication of a defining set of shallow-water model tests [6], which were instrumental in bringing the topic of ship squat to Ernie’s attention. In 1966, he published his groundbreaking article “Shallow-water flows past slender bodies” [9]. For Ernie, this was a fairly straightforward extension of his earlier deep-water work [8]. However, this new method produced a closed solution to the dynamic sinkage and trim of a ship in shallow water, which would form the basis of most such methods in use today.

The first assumption of the method was that the flow is incompressible, a natural consequence of the very low Mach numbers at which ships operate. The next assumptions were that the flow is inviscid and irrotational. These assumptions were based on the fact that ships operate at extremely high Reynolds numbers (order 10^9) and are slender streamlined objects. Hence viscous effects are confined to a thin boundary layer near the ship’s hull, and do not significantly affect the pressure distribution around the hull, except possibly at the stern. Therefore pressure distributions calculated using such methods should be adequate for predicting overall quantities such as sinkage and trim.

Based on these assumptions, a velocity potential would exist, and would satisfy Laplace’s equation throughout the fluid domain, subject to appropriate boundary conditions. The ship-fixed coordinate system we shall hereafter use to describe the flow around a ship is shown in Figure 2. The third dimension z is centred on the undisturbed free surface, positive upwards.

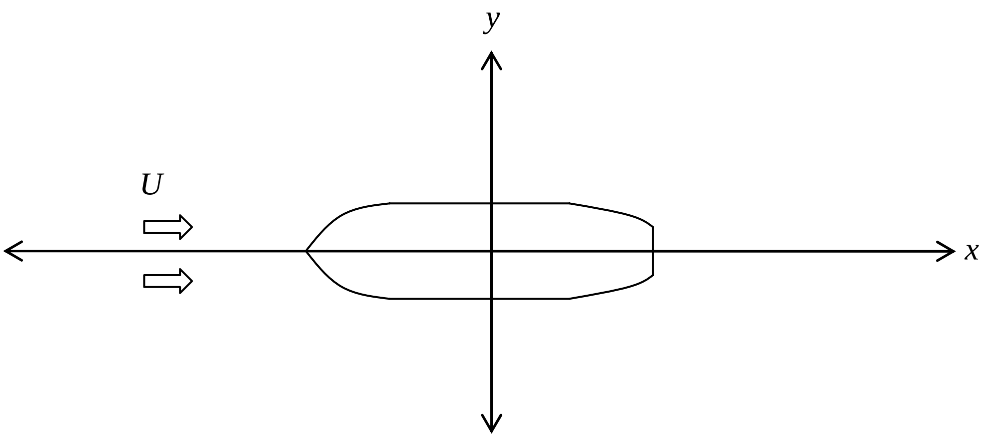

Figure 2: Plan view of ship-fixed coordinate system

ϕ is defined hereafter as the disturbance velocity potential, so that the total fluid velocities in the ship-fixed frame are given by (grad$(Ux+\phi)$).

For flow past a slender object, it was already well known [10] that the kinematic and dynamic boundary conditions on the free surface could be linearized and combined to give

$$g\frac{\partial\phi}{\partial z}+U^2\frac{\partial^2\phi}{\partial x^2}=0 \quad \text{on} \quad z=0 \tag{1}$$

This formed the boundary condition on the free surface. In open water of constant depth *h*, the boundary condition on the seabed was the kinematic condition

$$\frac{\partial\phi}{\partial z}=0 \quad \text{on} \quad z=-h \tag{2}$$

Tuck's 1966 article combined the ideas of shallow-water flows and slender-body flows. He made the realistic assumption that, for a ship in sufficiently shallow water that it might be at risk of grounding, the ship beam, ship draught and water depth were all of similar order ε and small compared to the ship length *L*. The first consequence of these assumptions, when applied to the governing Laplace equation and boundary conditions (1,2), was that the flow around the ship was nearly horizontal, with the velocity potential satisfying, to leading order,

$$\left(1-F_h^2\right)\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}=0 \tag{3}$$

This equation had been obtained by Michell [11] for a thin vertical strut extending from bottom to top of a shallow stream. For a ship hull, there remained the problem of defining the kinematic boundary condition on the hull, which Ernie solved using matched asymptotics, in a similar way to the deep-water problem [8]. An "outer region" was defined for $y=O(L)$, where the flow was governed by equation (3). An "inner region" was defined for $y=O(\varepsilon)$, in which the leading-order velocity potential was a function of x only, while the second-order flow was a function of (y,z) only. The second-order flow satisfied a rigid free surface boundary condition, and produced an outflow at the outer limit of the inner region, which was constant in the vertical. This transverse outflow was matched to the inner limit of the outer region, to yield a hull boundary condition which could be used for the outer flow, namely

$$\frac{\partial\phi}{\partial y}=\pm\frac{U}{2h}\frac{dS}{dx} \quad \text{on} \quad y=0_{\pm} \tag{4}$$

As far as the outer flow is concerned, the ship behaves like a line of sources in the (*x,y*) plane, with source strength proportional to the rate of change of ship cross-sectional area at each station *x*. The velocity potential for subcritical flow was therefore solved to be

$$\phi = \frac{U}{4\pi h\sqrt{1-F_h^2}} \int_{-L/2}^{L/2} \frac{dS}{d\xi} \ln\left[(x-\xi)^2 + \left(1-F_h^2\right)y^2\right] d\xi \quad (5)$$

Hydrodynamic pressure is found using Bernoulli's equation, and the pressure beneath the hull found by taking the limit as $y \to 0$ (the inner limit of the outer solution). Sinkage and trim then follow by hydrostatics. The symmetry of equation (5) means that for a fore-aft symmetric hull, sinkage is non-zero and trim is zero at subcritical speeds. A similar version of equation (5) exists for supercritical speeds [9], which predicts that trim is non-zero and sinkage is zero for fore-aft symmetric hulls.

3 Simplifying the results

Prof. Tuck recognized that his slender-body shallow-water results would be of little use to mariners in its current form. To quote from [12],

"*It must be confessed that all these results are of little direct benefit to the navigator, as complex mathematical computations are required in order to estimate the squat in any given situation. Such computations can be readily performed by suitably qualified persons with access to a computer, but hardly by an individual pilot in his line of duty!*"

In order that his results may be more readily accessible to mariners, Tuck performed a dimensional analysis of his sinkage expression, and found that it could be written in the following form [13]:

$$s_{\mathrm{LCF}} = c_s \frac{\nabla}{L^2} \frac{F_h^2}{\sqrt{1-F_h^2}} \quad (6)$$

The sinkage coefficient c_s then satisfies

$$c_s = \frac{L^2}{2\pi A_{\mathrm{WP}}\nabla} \int_{-L/2}^{L/2} \int_{-L/2}^{L/2} \frac{dS}{d\xi} \frac{B(x)}{x-\xi} d\xi\, dx \quad (7)$$

This was calculated for a range of example hulls, and found to be approximately constant (between 1.3 and 1.5) in all cases. A value of 1.5 was suggested as a conservative figure for general use.

Tuck also recognized that in most practical cases, the depth Froude number is low and $\sqrt{1-F_h^2}$ could be replaced by unity, yielding the simpler approximation [12,13]

$$s_{\mathrm{LCF}} = 1.5 \frac{\nabla}{L^2} \frac{U^2}{gh} \quad (8)$$

The articles [12,13] do not mention dynamic trim, despite a valid theoretical formula for this having been developed in [9]. The present author believes that there are three reasons for this:

1. Ships built up until the 1960s generally had their centre of buoyancy close to midships, in which case the dynamic trim is small. Prof. Tuck had most likely done calculations for example ships and found that midship sinkage, rather than dynamic trim, was the dominant effect. To quote from [1], "*.. the term 'squat' includes both sinkage… and trim… (for) subcritical speeds, sinkage is the more important phenomenon, because the increases in draft at bow and stern are large compared with the differences between them.*"

2. Dynamic trim is strongly affected by the shape of the hull, so the formula for dynamic trim cannot be simplified as much as that of sinkage.
3. Flow near the stern of the ship is influenced by possible flow separation, as well as the ship's propeller, neither of which are included in the theory [9]. These factors influence dynamic trim far more than midship sinkage. It is probable that Prof. Tuck understood these limitations of the dynamic trim theory from his time at the David Taylor Model Basin, and preferred to concentrate on the more accurate midship sinkage predictions.

Despite Prof. Tuck not developing a dynamic trim formula for general use, other researchers [14,15] were able to analyze his 1966 trim formulae in a similar manner to that which was done for sinkage. This allowed the dynamic trim angle to be written

$$\theta = c_\theta \frac{\nabla}{L^3} \frac{F_h^2}{\sqrt{1-F_h^2}} \tag{9}$$

Unlike for sinkage, the coefficient c_θ depends strongly on the hull shape, being zero for fore-aft symmetric hulls, positive for modern bulk carriers with centre of buoyancy well forward, and either positive or negative for containerships. Following [9], the trim coefficient can be calculated from

$$c_\theta = \frac{-L^3}{2\pi I_{LCF} \nabla} \int_{-L/2}^{L/2} \int_{-L/2}^{L/2} \frac{dS}{d\xi} \frac{(x - \mathrm{LCF})B(x)}{x-\xi} \, d\xi \, dx \tag{10}$$

Therefore Prof. Tuck had laid the foundations for both a complicated and a simple method to predict ship squat in open water of constant depth. For mathematicians capable of evaluating singular double integrals, the sinkage and trim coefficients could be calculated for any given hull shape using equations (7,10). For model testing organizations, the coefficients could be calculated empirically for each hull using equations (6,9) and model test results. Once the coefficients had been calculated theoretically and checked empirically, they could be generalized for different hull types (e.g. bulk carriers, containerships) for use in equations (6,9) by mariners.

4 Ships in canals and dredged channels

The obvious extension of Tuck's 1966 open water theory was to include laterally restricted water. Tuck analyzed the case of a ship travelling along the centreline of a canal [16], and found that the governing equation (3) was best solved by Fourier transform. The open water case also had a Fourier-transform type solution, though this is more computationally intensive than the original source-type solution put forward in [9]. Nevertheless, Tuck must have investigated the open water Fourier-transform solution in stating that the canal solution and the open-water source solution agree in the wide channel limit [16].

A one-dimensional hydraulic theory for ships in very narrow canals, which involves solution of a cubic equation for the local flow velocity, was already well-known at this time [17,18]. Tuck included in his 1967 paper a simple linearized hydraulic theory for flow past a ship in a narrow canal. The general width solution was shown to agree with the hydraulic theory in the narrow-channel limit.

An important result from the general-width theory [16] was that the influence of canal width depends primarily on (width / shiplength) ratio, rather than (width / beam) ratio. Canal walls first start to increase the squat when the width is around 3 times the shiplength. When the width is equal to the shiplength, the sinkage and dynamic trim at low speed are around 30% larger than in open water.

The next result was that the effect of canal walls becomes increasingly important as the ship speed increases. In fact, Tuck found that the percentage increase of sinkage and trim in a canal over the open water values is almost a universal curve for all ship types, depending on the width parameter

$$\overline{w} = \frac{w}{L}\sqrt{1-F_h^2} \qquad (11)$$

As well as wall-sided canals of constant depth, Tuck also investigated dredged channels, involving a step depth change to shallower water on either side of the deep central channel [19]. The problem was solved by Fourier transform and sample numerical results given. An interesting case studied was when the flow in the deep channel is subcritical, but in the shallower water is critical, in which case there is zero flux from the deep to the shallow water, and the flow past the ship is the same as if it were in a wall-sided canal.

5 Transcritical flow in open water

Prof. Tuck recognized that his open water theory became singular at the "critical speed" $F_h = 1$. For a ship travelling at the critical speed, i.e. the natural speed of long waves in shallow water, there is no restoring force on the free surface according to linear theory, as any wave shape can travel at the same speed as the ship. Therefore the free surface is undefined according to linear theory. From [9],
"*It is clear that good agreement cannot be expected near to the critical speed, where the first-order theory predicts infinite values for all forces. By analogy with the aerodynamics of transonic flow, we should expect that in order to predict correctly the finite values obtained in this region we should need to consider some special nonlinear effects, and this will not be done here.*"

Despite the problems at the critical speed, the behaviour of the open water theory at subcritical speeds was correct: a rapid increase in dynamic sinkage as the critical speed was approached. Tuck knew from the aerodynamic analogy that travel at the critical speed should be perfectly possible, and had studied closely the model test results of Graff, Kracht & Weinblum [6] showing the sudden changes in sinkage and trim that occur close to the critical speed. Early in this author's PhD, Ernie brought out a carefully-guarded original large-scale set of the 1964 model test plots, clearly showing the large peaks in sinkage, trim and wave resistance, and said, "*We need to work out what's going on here*". We then set about including the leading-order effect of dispersion in the governing equation (3), and solving the resulting equation by Fourier transform. When unsure about the formulae for Filon quadrature coefficients given in a text book, Ernie declared, "*We'll have to work it out for ourselves. I'll race you*." He won, and we went on to develop a computational method for predicting maximum sinkage and trim [20], following on from the finite-depth method [13] developed by Tuck & Taylor in 1970.

Other recent research [21] includes nonlinear terms in the transcritical flow equations, and calculates the unsteady soliton production of a ship started impulsively from rest. This method shows promise for predicting the effects of nonlinearity on transcritical sinkage and trim in open water.

6 Transcritical flow in a canal

One of the results of nonlinear hydraulic theory [18] was that there existed a range of ship speeds in a canal for which no steady flow was possible. Tuck & Taylor [13] quantified this speed range in terms of the ship and canal cross-sectional areas, producing a diagram of the speed limits of steady subcritical and supercritical flow.

By combining unsteadiness, nonlinearity and two-dimensionality for canal flows, modern researchers [22,23] have studied the interesting problem of unsteady soliton production in the transcritical speed range, as well as further refining the speed limits of steady subcritical and supercritical flow.

7 Accuracy of Tuck's squat formulae

Model test results

Tuck's 1966 paper [9] included a comparison of his theoretical predictions with the measured sinkage and trim [6] of a Taylor A3 (frigate-type) hull. Perhaps unfortunately, the independent model test results had concentrated on the transcritical speed range, and did not include accurate results for low speeds, at which Tuck's 1966 theory is most accurate and practically useful. Nevertheless, Tuck showed the comparison over the full experimental speed range $0.5 < F_h < 1.5$, including the theory's known singularity at $F_h = 1$. It was shown that sinkage and trim were quite well predicted at the lower subcritical speeds as well as higher supercritical speeds.

We shall now compare the predictions from Tuck's theory with some recent model test results for a bulk carrier hull at realistic (low) speeds. Figure 3 shows a comparison between measured sinkage coefficients [24] for a MarAd L-Series bulk carrier model, and predictions from Tuck's theory [16]. The canal theory has been used because of the finite width of the testing tank ($w/L = 2.1$) . In this case the trim coefficient is approximately the same as it would be in open water, but the sinkage coefficient is 9% higher.

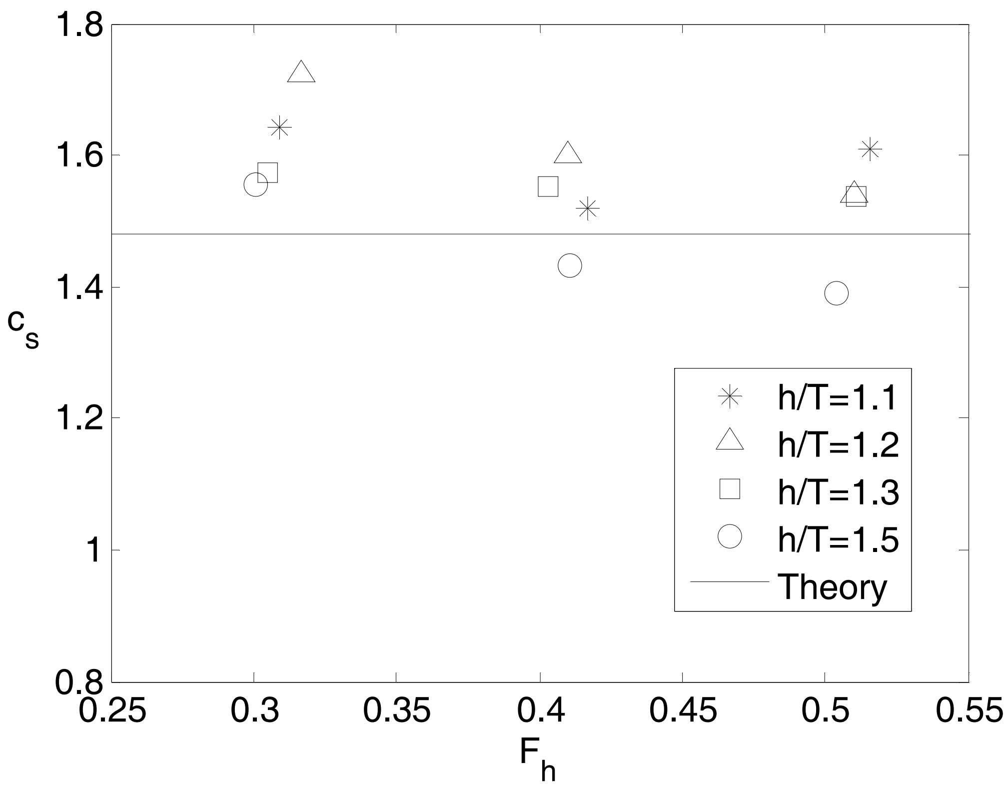

Figure 3: Measured and predicted sinkage coefficient for a bulk carrier model

The comparison shows us that:

1. The general dimensional nature of the solution is accurate, with no clear influence of either speed or depth/draught ratio on the measured sinkage coefficient.
2. The measured sinkage coefficient is very similar to that predicted by Tuck's theory.

There is a common misconception in the maritime community that squat is the result of water being accelerated as it passes underneath the ship, so that as the clearance becomes small, the squat becomes catastrophically large. The test results above show that there is no such effect at small h / T, other than the normal sinkage increase as the water depth decreases. In fact, as the clearance becomes small, more water is simply diverted around the sides of the ship. To quote from [9], "*One might expect a very large velocity and hence abnormally low pressure at any point where a cross-section almost touches bottom, but the conclusion from the present analysis is that to first order the pressure at such a point is no lower than anywhere else on the same cross-section. Presumably this implies that the fluid passes to the side of any such close gap so as to keep the velocity there comparable with that elsewhere on the cross-section.*"

Figure 4 shows dynamic trim measurements and predictions for the same bulk carrier hull.

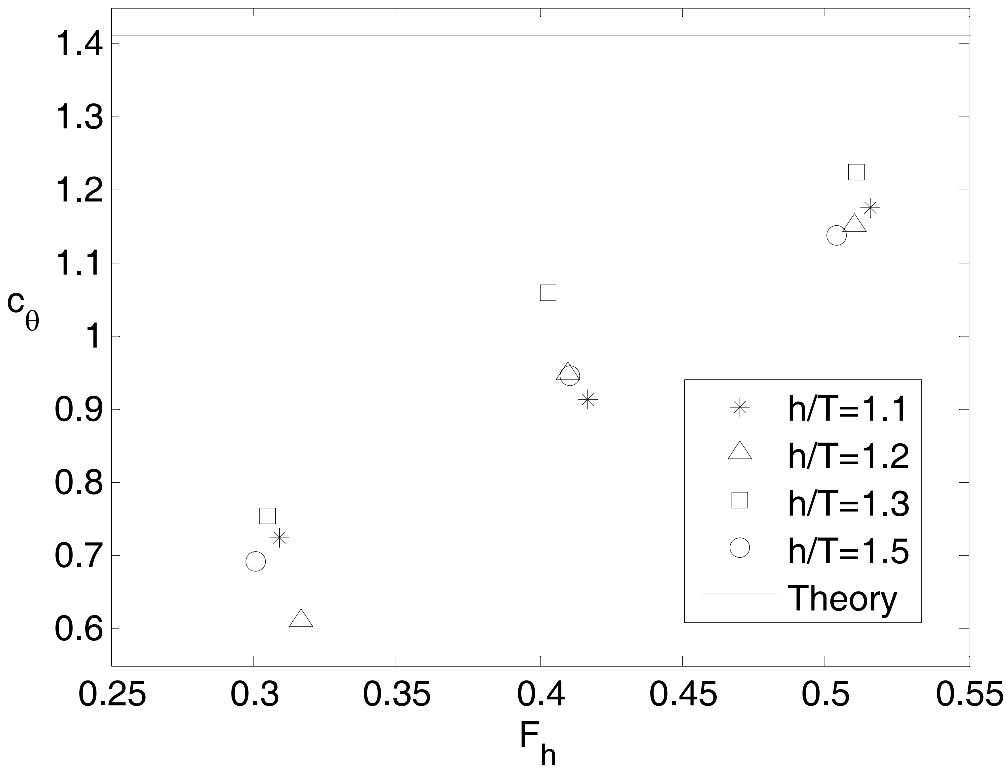

Figure 4: Measured and predicted trim coefficient for a bulk carrier model

We see that the dimensional formula (9) accurately predicts the effect of depth-draught ratio, with no clear influence on measured trim coefficients at constant speed. However, the effect of speed is not accounted for correctly, and the measured trim is always less bow-down than predicted. The model tests referred to used a towed rather than self-propelled model; this fact, as well as the low Reynolds number of the model scale flow, make it likely that flow separation was occurring at the stern, decreasing the bow-down trim in the model tests. This effect would be speed-dependent.

Full-scale results
Since the advent of real-time kinematic GPS, ship squat can now be measured accurately at full scale. Squat measurements of containerships are described in [25], including comparison with Tuck's open water theory. It was seen that LCF sinkage was well predicted by the theory, except in cases of rapid depth or speed changes, which violate the steady assumption. Dynamic trim was also well predicted by the theory along the straight sections of the route. The present author has also undertaken full-scale ship squat trials on bulk carriers in Torres Strait (not yet published), showing good agreement with Tuck's open water theory.

Another set of full-scale ship squat trials is described in [26], including comparison of bow sinkage with the ICORELS formula (see Section 7), which is based on Tuck's dimensional formulae (6,9). This comparison showed very good agreement.

8 Use of Tuck's formulae in modern ship squat prediction
The 1997 PIANC guidelines “Approach channels: a guide for design” [27] gives an overview of the eleven most accurate practical squat prediction methods in use at that time, for open water or moderate-width canals. The PIANC guidelines begin by saying that the fundamental theoretical study into squat was that of Tuck in 1966. Four of the eleven models [14,15,28,29] are directly based on Tuck's dimensional equations (6,9), using empirically-determined coefficients. These include the ICORELS model [29] developed by PIANC as the most appropriate for general prediction of bulk carrier squat.

As well as the squat models mentioned in [27], other authors [30,31,32] have also developed specialized squat models for open or confined water, based on Tuck's method.

9 Other modern ship squat prediction methods
Apart from the Tuck-based methods, the other practical ship squat prediction methods recommended in the PIANC guidelines [27] are all regression formulae. These are best-fit equations to model testing data, using perceived input variables, but generally no physical basis. Such regression methods clearly work well for the types of ships used to develop the formulae, but may give erratic results for different ship types, especially those lying outside the range of parameters used for the model tests.

An alternative theoretical approach to ship squat prediction in open water is the directed fluid sheet method [33]. Whereas Tuck's method [9] models the flow as predominantly longitudinal and transverse (x, y), the directed fluid sheet method models the flow beneath the ship as predominantly longitudinal and vertical (x, z). This method assumes a large (beam/draught) ship and models the flow beneath the centreline of such a ship.

The use of panel methods is becoming increasingly common for predicting ship squat in open or confined water. Example calculations using a Rankine-source panel method are described in [34].

Note that all squat methods which use the full ship hull shape are subject to the same difficulties in routine practical application as Tuck's original method; this will be discussed in the following section.

10 Calculating Tuck's sinkage and trim coefficients

Model tests give a convenient way of determining the sinkage and trim coefficients for a given hull, which can then be assumed the same at full-scale, as done by the empirical methods described in Section 8. However, there remains the question of viscous scale effect with respect to dynamic trim, especially if a towed rather than self-propelled model is used. It is expected that the trim coefficient equation (10) will be more accurate at full scale than at model scale because of the much higher Reynolds number and therefore lesser effect of viscosity. Preliminary investigations by the author on containerships in Hong Kong [25] and bulk carriers in Torres Strait suggest that this is the case. Therefore it is still desirable to be able to calculate the sinkage and trim coefficients for a given hull theoretically.

As Prof Tuck recognized, equations (7,10) for determining exact sinkage and trim coefficients are limited to practising mathematicians. Use of the formulae is also thwarted by the difficulty in obtaining the section area curve $S(x)$ and waterline breadth curve $B(x)$ for the ship in question. This information is normally obtained from the "body plan", a design drawing showing transverse sections of the hull at different longitudinal spacings. Alternatively, the information can be obtained from a Table of Offsets, which gives (x,y,z) coordinates of the hull surface.

An example body plan is shown in Figure 5, with the rear half of the hull on the left and forward half (including bulb) on the right.

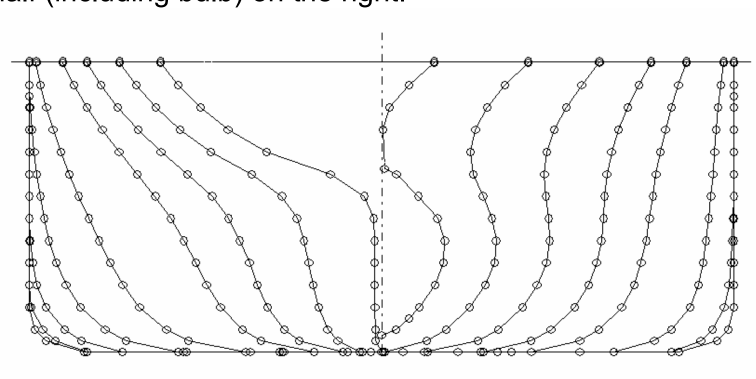

Figure 5: Example body plan for a bulk carrier hull

Unfortunately, body plans and offset tables are confidential for newer ships, as they define the entire shape of the hull. For older ships where hull confidentiality is no longer an issue, such information is generally not carried onboard the ship; instead, it is kept in the original design office, which may no longer exist. Therefore it is not practical to routinely obtain full $S(x)$, $B(x)$ curves for normal commercial ships.

A good source of readily-available hull data for commercial ships is the Trim and Stability Book, which is kept onboard all large vessels. This book lists many important waterplane and volume parameters, over a full range of midship draughts and static trim values. A method will now be described for approximating the $S(x)$, $B(x)$ curves for a bulk carrier hull, based on information found in its Trim and Stability Book.

Firstly, a representative hull is chosen, with similar proportional bulb length to the bulk carrier being modelled. The hull should also be fairly similar in terms of the other dimensionless parameters, so that only minor modifications are required. A range of representative bulk carrier hulls is publicly available, including systematic series [35,36] and representative hulls found in naval architecture software such as Maxsurf™ and Seaway™.

The representative hull is modelled at the same draught (as a proportion of its full load draught) as the ship, and the same static trim. It is scaled to the same length, beam and draught as the ship we are modelling. The waterline breadth and section area curves are then found for the representative hull. These may be non-dimensionalized, as the sinkage and trim coefficient equations (7,10) depend only on the shape of the ship and not the length, beam or draught. However, the author is reminded of Prof. Tuck's plea [8] to avoid non-dimensional variables when developing equations, and we shall continue here in dimensional variables.

Example $B(x)$ and $S(x)$ curves are shown in Figure 6 for a Japan standard series 1704B hull with bulbous bow [36].

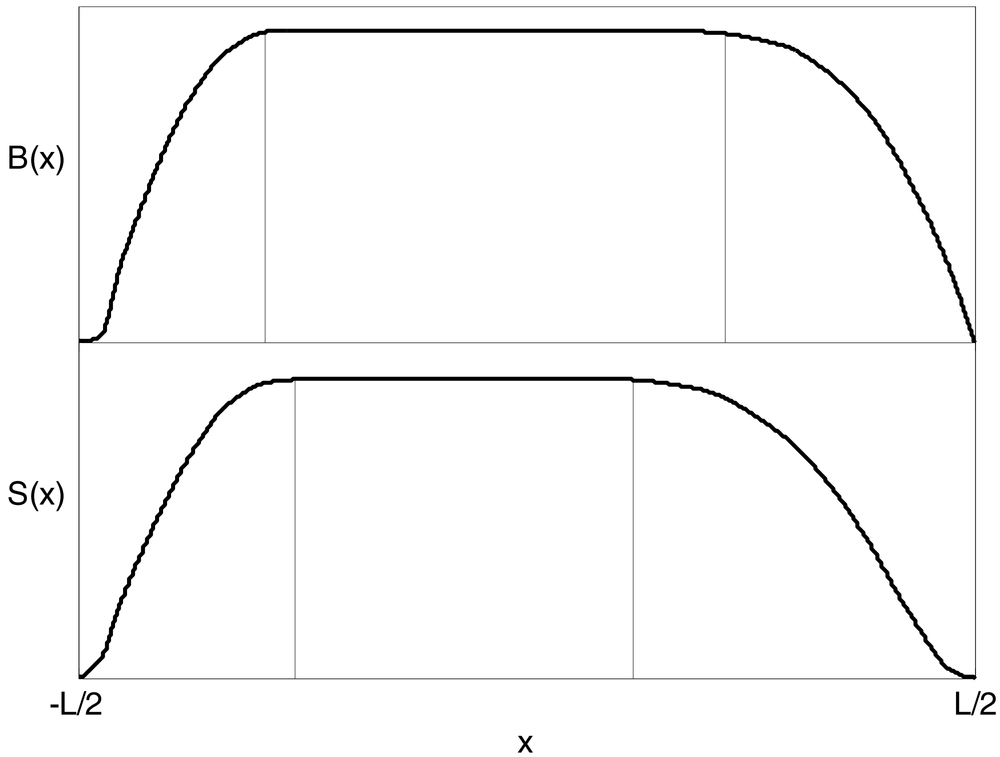

Figure 6: *B*(*x*) and *S*(*x*) curves of standard series bulk carrier hull [36], showing limits of parallel midbody. Front of bulb at $x = -L/2$, stern at $x = L/2$.

We can see that a bulk carrier hull is very block-like, with both the waterplane and the section area distribution being characterized by a long parallel midbody. By adjusting the limits of this parallel midbody, while keeping the bow and stern curve shape the same, we can adjust the $B(x)$, $S(x)$ curves so that they better represent the ship we are modelling.

The sinkage and trim coefficient equations (7,10) depend on the *shape* of the ship rather than the length, beam or draught. For a given length, beam and draught, this shape can be characterized principally by the ship's displaced volume ∇, longitudinal centre of buoyancy (LCB), waterplane area A_{WP} and longitudinal centre of floatation (LCF). All of these quantities can be found from the Trim and Stability Book at the appropriate draught and static trim.

In order to model the correct waterplane area and LCF, we can adjust the forward and aft limits of the waterplane's parallel midbody shown in Figure 6. The method is identical for the displaced volume and LCB, by adjusting the forward and aft limits of the section area parallel midbody.

Here we shall describe the method for the waterplane, based on simple geometrical considerations. The forward and aft limits of the representative hull's parallel midbody lie at x_{fwd} and x_{aft}, and the forward and aft waterplane areas (up to the parallel midbody) are A_{fwd} and A_{aft}. In order to get the waterplane area and LCF correct, we stretch the forward and aft curved portions longitudinally by factors α_{fwd} and α_{aft}, keeping the bow and stern in the same position. The forward end of the parallel midbody is therefore moved to $-L/2+\alpha_{\mathrm{fwd}}(x_{\mathrm{fwd}}+L/2)$ and the aft end to $L/2-\alpha_{\mathrm{aft}}(L/2-x_{\mathrm{aft}})$. The new waterplane area is therefore

$$A_{\mathrm{WP}} = \alpha_{\mathrm{fwd}}A_{\mathrm{fwd}} + \alpha_{\mathrm{aft}}A_{\mathrm{aft}} + \left[L-\alpha_{\mathrm{aft}}(L/2-x_{\mathrm{aft}})-\alpha_{\mathrm{fwd}}(x_{\mathrm{fwd}}+L/2)\right]B_{\mathrm{m}} \qquad (12)$$

If the longitudinal centres of floatation of the forward and aft curved sections are at ξ_{fwd} and ξ_{aft} for the representative hull, the new longitudinal centre of floatation (LCF) satisfies

$$\begin{aligned}A_{\mathrm{WP}}\mathrm{LCF} &= \alpha_{\mathrm{fwd}}A_{\mathrm{fwd}}\left[-L/2+\alpha_{\mathrm{fwd}}(\xi_{\mathrm{fwd}}+L/2)\right]+\alpha_{\mathrm{aft}}A_{\mathrm{aft}}\left[L/2-\alpha_{\mathrm{aft}}(L/2-\xi_{\mathrm{aft}})\right]\\ &+\left[L-\alpha_{\mathrm{aft}}(L/2-x_{\mathrm{aft}})-\alpha_{\mathrm{fwd}}(x_{\mathrm{fwd}}+L/2)\right]\left[\alpha_{\mathrm{fwd}}(x_{\mathrm{fwd}}+L/2)-\alpha_{\mathrm{aft}}(L/2-x_{\mathrm{aft}})\right]B_{\mathrm{m}}/2\end{aligned} \qquad (13)$$

Equations (12,13) can now be solved to determine the required scaling factors α_{fwd} and α_{aft}, so that the correct waterplane area A_{WP} and longitudinal centre of floatation LCF are achieved. Once this is done, the representative hull's $B(x)$ curve shown in Figure 6 is modified by longitudinally stretching the forward section by α_{fwd} and aft section by α_{aft}.

A similar process is followed with the section area curve as described above. The $B(x)$, $S(x)$ curves thus obtained can then be input into equations (7,10) to determine the sinkage and trim coefficients for that particular bulk carrier hull.

For a containership, a similar method can be employed to obtain the correct displaced volume, LCB, waterplane area and LCF, based on a chosen representative hull. The difference is that containerships do not have a long parallel midbody, so the required changes must be achieved by smoothly filling out and longitudinally shifting the waterplane to achieve the correct waterplane area and LCF, for example. Parametric transformations are available in naval architecture software (e.g. Maxsurf$^{\mathrm{TM}}$) to achieve this.

11 Conclusions

The contributions of Prof. E.O. Tuck to the field of ship squat prediction have been highlighted. His important research in this field included the development of:

- matched asymptotic expansions for ship hulls [8] in 1964
- a slender-body shallow-water method for ships [9] in 1966
- extension of the theory to canals [16] in 1967 and dredged channels [19] in 1975
- making the formulae accessible to mariners and model testing organizations through an accurate dimensional representation with approximate coefficients [12] in 1970

- a computational method to accurately predict maximum sinkage through the critical speed [20] in 2001

The widespread use of Tuck's formulae in modern ship squat prediction has been described, including its adoption by PIANC [27] as the method of choice for bulk carrier hulls. Tuck's formulae and its derivatives have been shown to be in good agreement with model tests and full-scale tests for bulk carriers and containerships.

A practical problem with the application of Tuck's original formulae (6,7,9,10) to commercial ships is that it is normally not feasible to obtain their exact $B(x), S(x)$ curves. For this reason, a method has been described for approximating the $B(x), S(x)$ curves of a general bulk carrier or containership hull, based on information found in its Trim and Stability Book and a modified representative hull.

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