THE EFFECT OF HORIZONTAL REFRACTION ON BACK-AZIMUTH ESTIMATION FROM THE CTBT HYDROACOUSTIC STATIONS IN THE INDIAN OCEAN

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Abstract: *Both large-scale spatial variations of oceanographic characteristics and changes in the bottom topography can induce horizontal refraction. This study uses numerical modelling to investigate the horizontal refraction of low-frequency underwater sound propagating in the Indian and Southern Oceans, and the resulting effects on estimates of back-azimuths from the Comprehensive Test-ban Treaty (CTBT) hydroacoustic stations. It is shown that the horizontal wavenumbers of different modes in the deep water regions of the Southern Ocean within and beyond the Antarctic Convergence Zone (ACZ) have stronger horizontal gradients than those in the Indian Ocean. The deviation of bearing to the signal source from the two CTBT hydroacoustic stations off Cape Leeuwin (Western Australia) and off the Chagos Archipelago (BIOT) resulting from horizontal refraction does not exceed 0.20° in the most parts of the Indian Ocean. By contrast, strong horizontal wavenumber gradients in the high-latitude regions of the Southern Ocean and, especially, across the ACZ, cause the acoustic propagation path to deviate by as much as one degree from the geodetic line. This deviation depends strongly on the azimuth and range from the two CTBT hydroacoustic stations to signal sources located in the Southern Ocean beyond the ACZ.*

Keywords: *Horizontal refraction, Back-azimuth estimation, CTBT hydroacoustic station*

1. INTRODUCTION

Location of hydroacoustic events using back-azimuth of signal arrivals at the CTBT underwater listening stations is currently carried out ignoring possible effects of horizontal refraction due to large-scale spatial variations of the ocean environment [1]. In order to examine all possible errors of locating remote sources of underwater noise from the CTBT stations, it is necessary to investigate the effect of horizontal refraction on the estimates of bearing to noise sources from the CTBT receive triads. Mesoscale oceanographic features, such as eddies and internal waves can cause noticeable refraction at high and moderate frequencies [2-3]. At low frequencies, large-scale spatial variations of the sea depth and oceanographic characteristics can accentuate horizontal refraction for long-range acoustic propagation [4-6]. In this study, we followed the computational procedure proposed for the analysis of the Perth-Bermuda propagation experiment results [5]. It involves a combination of an adiabatic mode theory in the vertical dimension and a ray theory in the horizontal dimension and, therefore, takes into account horizontal refraction of individual modes due to both transverse sound speed gradients and bottom interaction over the continental slopes and sea mounts. The ray model was constructed on the surface of the Earth represented by an ellipsoid of rotation and expressed in terms of the latitude ϕ, longitude λ and azimuth angle α measured clockwise from the north. The ray equations on the Earth ellipsoid are [5]:

$$\dot{\phi} = \cos\alpha / \mu(\phi) \quad (1a)$$

$$\dot{\lambda} = \sin\alpha / v(\phi)\cos\phi \quad (1b)$$

$$\dot{\alpha} = \frac{\sin\alpha}{v(\phi)}\tan\phi - \left(\frac{\sin\alpha}{\mu(\phi)}\frac{\partial}{\partial\phi} - \frac{\cos\alpha}{v(\phi)\cos\phi}\frac{\partial}{\partial\lambda}\right)\log\kappa_n \quad (1c)$$

where k_n is the horizontal wavenumber of mode n. Overdot signifies the derivative with respect to arc length s. The variables μ and v are:

$$\mu(\phi) = r_{eq}(1-\varepsilon^2)/(1-\varepsilon^2\sin^2\phi)^{3/2}$$
$$v(\phi) = r_{eq}(1-\varepsilon^2\sin^2\phi)^{1/2} \quad (2)$$

and r_{eq} and ε are the equatorial radius and eccentricity of the Earth respectively. The last term in Eq. (1c) accounts for distortion of the ray paths due to the transverse gradients of the horizontal wavenumber k_n based on the Snell's law. If this term is neglected, the solutions of Eq. (2) are geodesics on the ellipsoid [7].

The modal horizontal wavenumbers were calculated using the KRAKEN program [8] on a horizontal grid with 0.1–degree grid size. The sound speed profiles were derived from the climatology salinity and temperature data gridded with 1-degree resolution in the World Ocean Atlas 2005 (http://www.nodc.noaa.gov/OC5/WOA05/pr_woa05.html). The bathymetry data were taken from the ETOPO2 Global 2-Minute Gridded Elevation Data (http://www.ngdc.noaa.gov/mgg/fliers/01mgg04.html). Both sound speed profiles and bathymetry were interpolated to 0.1-degree resolution to calculate modal wavenumbers. The system of ordinary differential equations in Eq. (1) can be solved using a 4-th or 5-th order Runge-Kutta method [9]. In the integration process, the maximum integration increment in distance was limited by the grid size and the modal wavenumbers were interpolated within the current grid cell in order to reduce errors of numerical integration.

2 SPATIAL VARIATIONS OF MODAL WAVENUMBERS

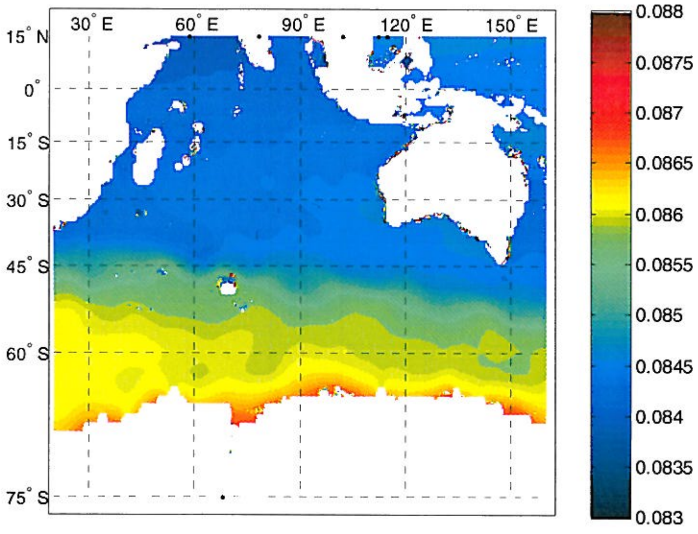

Figure 1. The map of mode 1 horizontal wavenumber at 20 Hz in the Indian and Southern Ocean region. Climatology data are taken for the winter season. The horizontal resolution is 0.1 °for both latitude and longitude.

Fig. 1 shows the variation of mode 1 horizontal wavenumber at 20 Hz over the region of the Indian and Southern Oceans in the winter season with 0.1 degree resolution. The spatial variation of horizontal wavenumbers is similar in general for different modes and at different frequencies. The Antarctic Convergence Zone (ACZ) is a transition area which divides the whole ocean region into the northern part with relatively low horizontal wavenumber values and the southern part with higher horizontal wavenumbers. In Southern Ocean region beyond the ACZ, the wavenumber increases with latitude. In the temperate ocean region above the ACZ, the modal wavenumber is relatively higher in the region south and southwest of Australia compared to the other regions. In the tropical ocean region, the modal wavenumber slightly decreases in the direction towards the north.

3 POTENTIAL EFFECT OF HORIZONTAL REFRACTION ON BEARING ESTIMATION

As follows from Eq. (1) the rate of azimuth deviation away from the local geodesic, governed by the last term of Eq. (1c), is proportional to logarithm of the transverse gradient of an equivalent refraction index $N_n = k_n / k_{n0}$, where subscript zero refers to an arbitrary reference value. In the deep water regions, the normal modes propagated over large distances are trapped either in the SOFAR channel in the temperate ocean or in the near surface channel in the high-latitude areas of the Southern Ocean and, therefore, the modal wavenumbers are dependent primarily on the sound speed profile rather than bathymetry. To examine the influence of wavenumber gradients, we selected two geodesic transects revealing the strongest wavenumber gradients in the two distinctive ocean environments: the deep water regions of the Indian Ocean and the Southern Ocean. The aim is to investigate possible ray distortion, which is assessed with respect to the rate of azimuth deviation from the geodetic line represented by the last term of Eq. (1c), for the rays which cross the transect in the perpendicular direction, and to analyze the potential effect of horizontal refraction on bearing estimation in these two regions. The first transect crosses the Indian Ocean from 45°S 90°E to 15°N 60°E and the second one crosses the Southern Ocean from 45°S 90°E to 68°S 75°E.

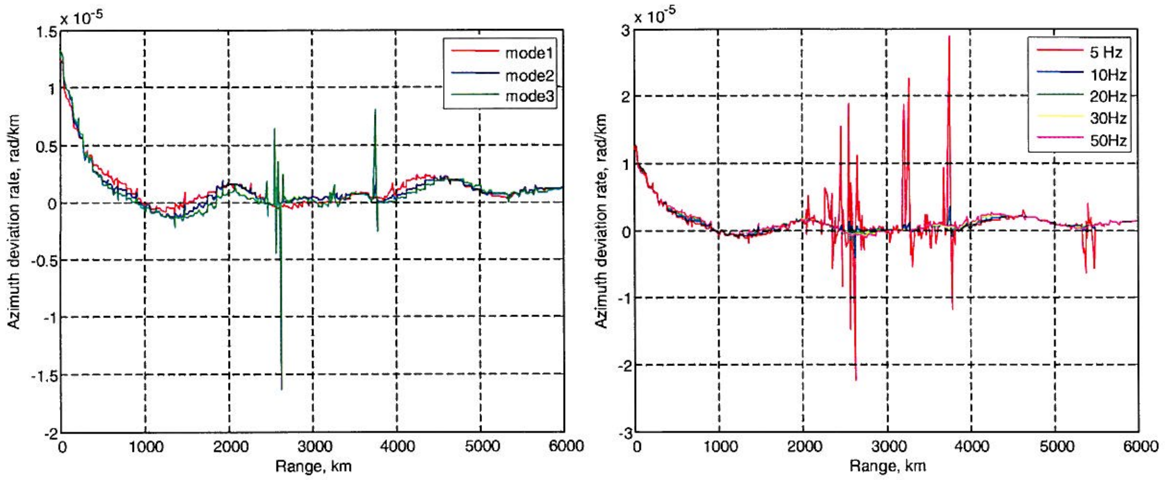

Figure2. The azimuth deviation rate of rays from the geodetic line due to the wavenumber gradient along the transect in the Indian Ocean (from 45°S 90°E to 15°N 60°E). Left panel shows the azimuth deviation rate for modes 1-3 at 20 Hz and the right panel shows the deviation rate for mode 1 at different frequencies.

The rate of azimuth deviation from the geodetic line for ray trajectories of different modes at different frequencies along the transect in the Indian Ocean is shown in Fig2. If considering only the effect of changes in the sound speed, the strongest azimuth deviation rate takes place in the beginning of the transect, where the sharpness of the sound speed minimum around the SOFAR channel axis decreases rapidly along the transect. For the rest of the region, the deviation rate is nearly zero. It is also seen in Fig. 2 that the ray deviation rate along this transect is almost independent of mode number and frequency for the modes trapped in the water column. For modes interacting with the seabed, e.g. mode 3 at 20 Hz and mode 1 at 5 Hz, the ray deviation rate varies significantly over the region where the interaction occurs.

As shown in Fig.3, the azimuth deviation rate along the transect in the Southern Ocean slightly decreases with mode number and increases with frequency. Such dependence takes place because higher order modes at lower frequencies penetrate deeper in the water column and hence are less sensitive to rapid change in the sound speed in the upper water layer across the ACZ. It can be noticed that the ray deviation rate across the ACZ and over the Antarctic continental slope is much higher than that along the transect in the Indian ocean, with a factor of around four for mode 1 at 20 Hz.

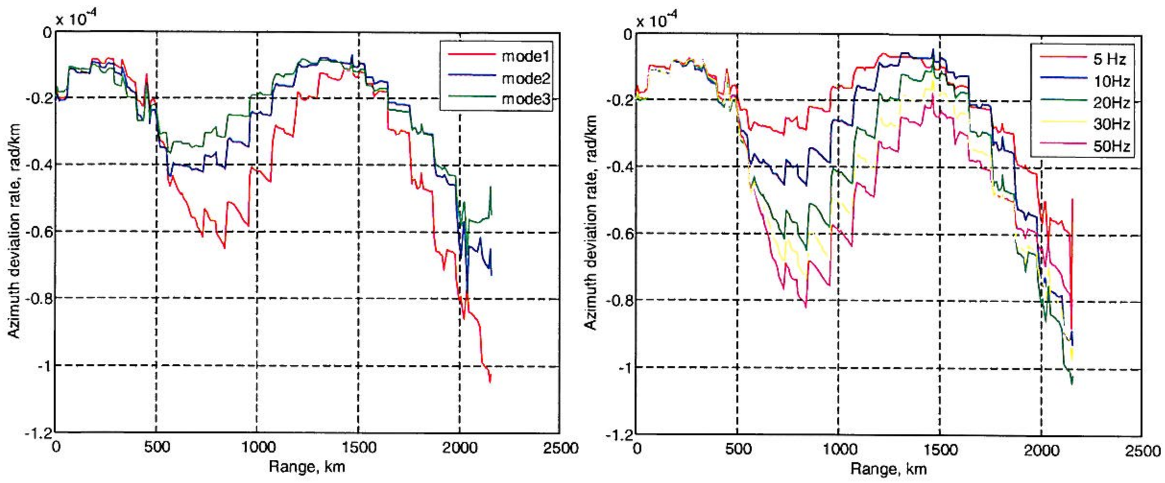

Figure3. The azimuth deviation rate of rays from the geodetic line due to the horizontal wavenumber gradient along the transect in the southern Ocean (from 45° S 90° E to 68° S 75° E). Left panel shows the deviation rate of the first three modes at 20 Hz and the right panel shows the same for mode 1 at different frequencies.

4 BEARING ERRORS DUE TO HORIZONTAL REFRACTION

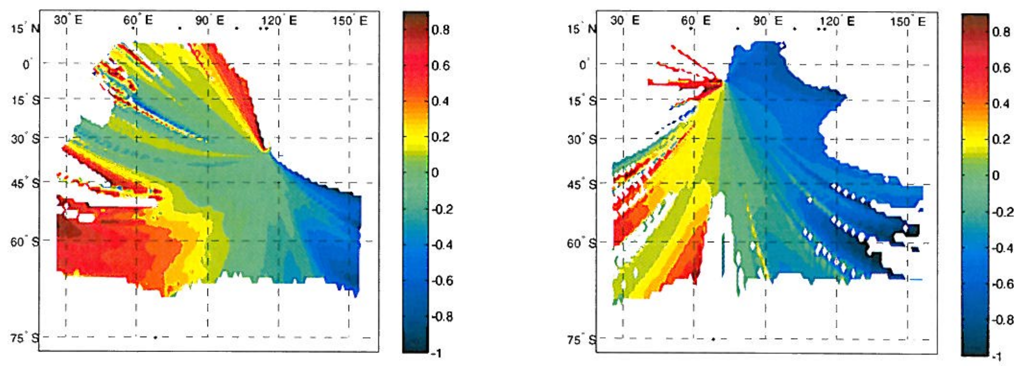

Figure4. The map of bearing deviation from the true azimuth observed from the HA01 (left panel) and H08S (right panel) stations for noise sources located in the Indian and Southern Oceans. The errors are due to horizontal refraction calculated for the spatial variations of mode 1 wavenumber at 20 Hz shown in Fig. 1.

Figure 4 shows the effect of horizontal refraction on bearing deviation from the true azimuth to sources of underwater noise observed from the HA01 and H08S stations, shown as a map of source location in the Indian and Southern Oceans. The deviation is calculated for the wavenumber of mode 1 at 20 Hz in winter shown in Fig.1. For the entire region, the bearing error due to refraction reveals strong azimuth and range dependence. The bearing deviation for HA01 does not exceed 0.2° for most parts of Indian Ocean region north of ACZ, except the shallow water regions over the continental slope and underwater mounts. When the source of noise is located beyond the ACZ in the Southern Ocean, relatively strong wavenumber gradient across the ACZ causes noticeable deviation of bearing to the true source location. The bearing errors due to refraction are less than 0.2° for the source location observed from HA01 at the azimuth of about 193^0 along which the propagation path is almost perpendicular to the ACZ frontal zone and therefore is negligibly affected by horizontal refraction. The absolute value of bearing errors due to refraction generally increases with azimuth moving away from this direction in both sides and reaches the maximum value of nearly 1° in the westernmost and easternmost parts of the southern Ocean observed fro HA01. For the H08S station, the propagation path to noise sources least distorted by horizontal refraction is along the azimuth of around 185^0. The variation of bearing errors to both sides from this azimuth is generally similar to that for HA01. The bearing errors for the northeast part of the Indian Ocean are considerable due to nearly perpendicular angles between the propagation path to H08S and gradients of the modal wavenumbers in this area.

4 CONCLUSIONS

In this paper, we investigated the effect of horizontal refraction on estimation of bearing to low-frequency sources of underwater noise observed in the Indian and Southern Oceans from the two CTBT hydroacoustic stations. The gradient of modal wavenumbers along two transects in the deep water regions of the Indian and Southern Ocean was used to examine the effect of horizontal refraction on azimuth deviation of the propagation path for different modes and frequencies. The study revealed that the effect of horizontal refraction can cause nearly ±1° errors of the back-azimuth estimation for sources of noise located in the Indian and Southern Oceans from both HA01 and HA08S CTBT hydroacoustic stations. The effect of horizontal refraction is highly dependent on the azimuth and range to the noise source.

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